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Pattern and Structure Assessment (PASA)

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Overview

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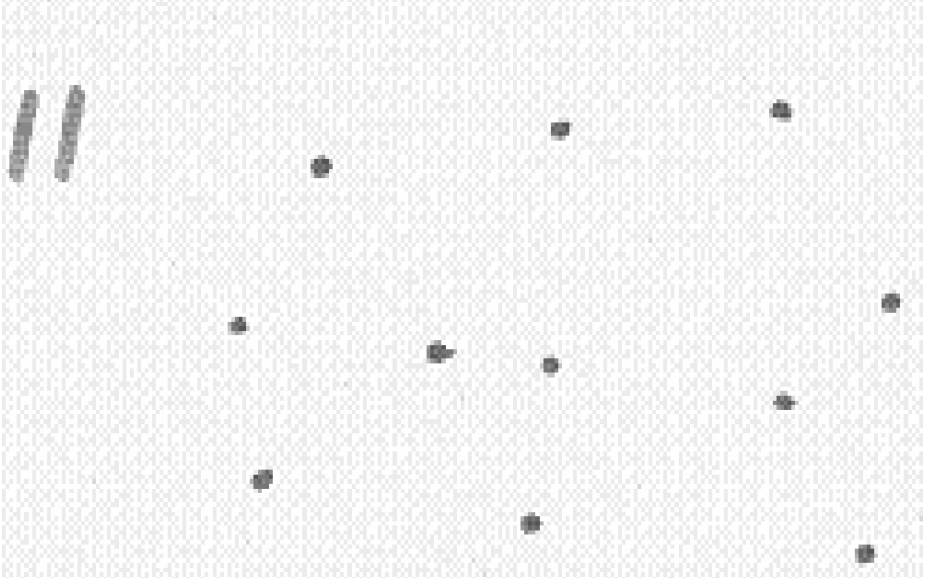
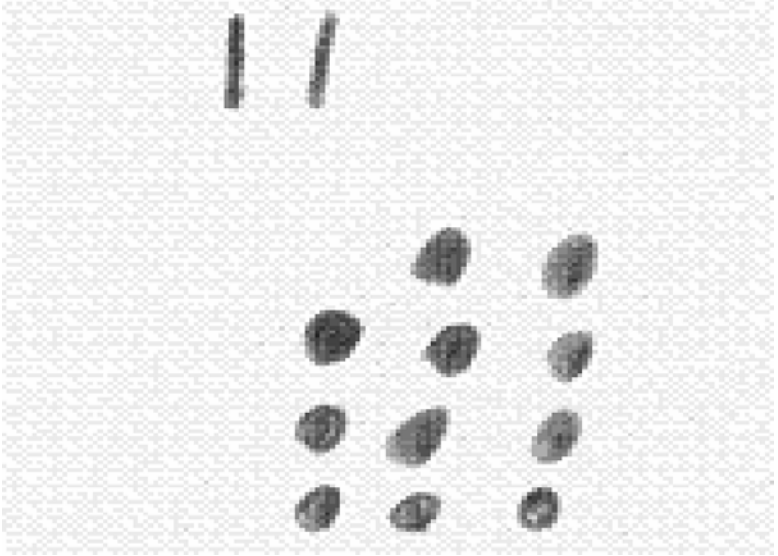
- What is mathematical pattern and structure?
- Awareness of Mathematical Pattern and Structure (AMPS)
- Background Research
- What is PASA: key features and components
- Scoring and Reporting
- AMPS scale
- PASA and curriculum outcomes
- Pattern and Structure Mathematics Awareness Program: PASMAP
- Where to from here?

Early underlying awareness of pattern and structure: Why? How?



“I see 11 all over as I count”



“I see 11 as a square of 3s and 2 more”

What is a Mathematical Pattern?

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- **Simple Repetition**: ‘unit of repeat’ ABC ABC ABC
- **Spatial Patterns**: 2D and 3D designs, tessellations, transformations
- **Growing Patterns**: increase or decrease systematically e.g. triangular number pattern 1, 3, 6, 10, 15
- **Functions**: relationship between variables e.g. table of values e.g. 1 dog - 4 legs; 2 dogs - 8 legs

What is Structure?

- Numerical structure: e.g. counting in multiples and equal groups
- Spatial structure: e.g. row and column array; similarity ‘same shape different size’ and congruence ‘same shape same size’
- Structure of units of measure
- Structural features that lead to abstraction and generalisation

  e.g. $a+b = b+a$

Background Research

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- Suite of studies 2002-2014 with 4 to 8 year olds
- Longitudinal evaluation of PASA
- Whole school assessment and intervention
- Case studies/ intervention special needs children
- Large longitudinal empirical evaluation NSW and Queensland
- 3- year longitudinal study of 24 gifted 5 year olds
- Study of statistical reasoning and pattern and structure
- Validation study ACER 2011-2105 (n=818)
- PASA and Pattern and Structure Mathematics Awareness Program: pilot studies in Europe, UK, New Zealand, US and Asia

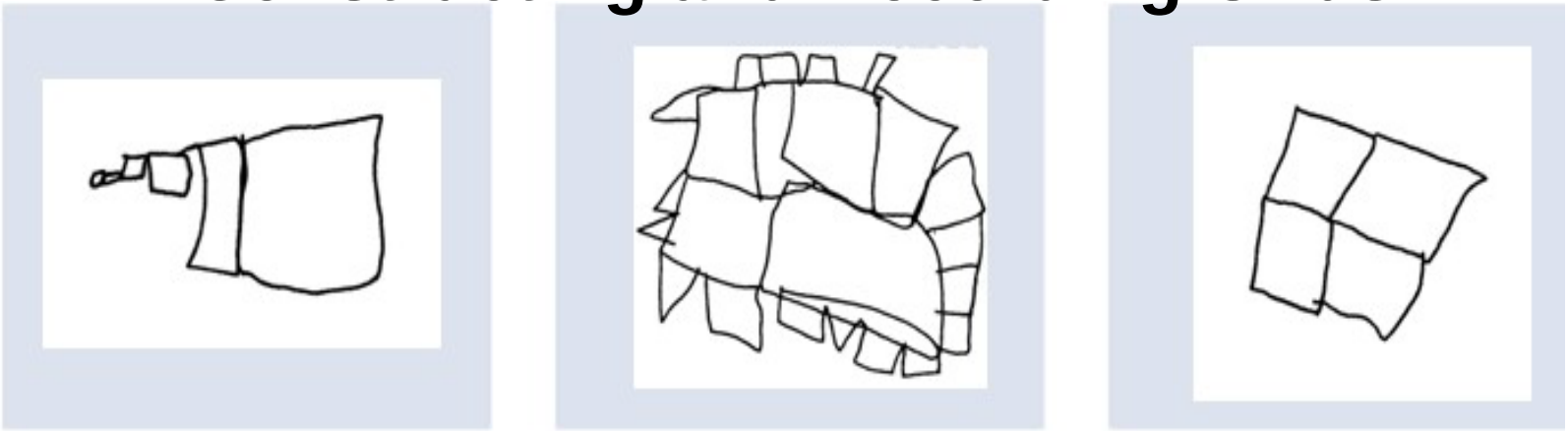
Awareness of Mathematical Pattern and Structure (AMPS): Research Findings

- Young children's use of mathematical pattern and structure underpins mathematical concept development and ability to abstract and generalise
- Particular spatial abilities (structuring) linked with mathematical development and linked with development of number
- The general underlying construct is called Awareness of Mathematical Pattern and Structure (AMPS)
- AMPS has two components: recognition of common structures and a tendency to look for patterns
- Early school mathematics achievement is highly correlated with children's level of AMPS
- Young children are far more capable than previously considered: capable of 'emergent' mathematical generalisation
- Huge range of capabilities in AMPS at any one age level

Kindergarten-Year 1 Study

- To validate a new conceptual framework for mathematics learning based on the development of AMPS
- Longitudinal evaluation of a school-entry mathematics program using classroom observations, interview-based assessment (PASA) and (ICDM) assessment;
- Provide basis for describing early algebraic reasoning
- Findings: children in PASMAP program significantly outperformed those in ‘regular’ program end of Year 1.

Tracking Individual Structural Development
Constructing and Recording Grids

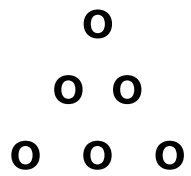


"I made them the same…the squares have the same on each side. It doesn't matter if they are big or small, they got the same sides. You have to put only the squares that you need. They have to be same size… I know they have to match if they are on top and on top you know like we made with the puzzles… I made a four with two and two…you can go both ways ".

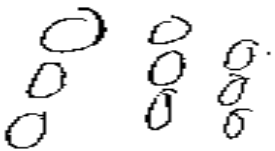
Focus on spatial structuring, doubling using collinearity (2 dimensionality), recognition of part/whole relationship, congruence and similarity, 'unit of repeat' / composite units

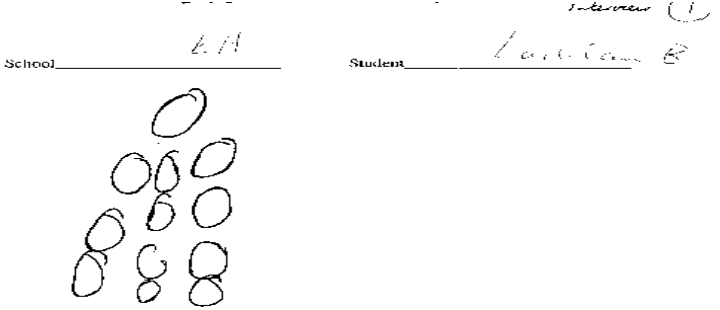
Triangular Array Task

(Flash card with pattern) Under this card is a pattern of dots. I'm going to show it to you for a short time. Then I want you to draw it. Draw exactly what you saw. What do you think comes next? Draw it here.



Triangular Array
Responses over 18 months: F-Year 1





Pattern and Structure Assessment (PASA)

An assessment program for early mathematics (F–2)

What is PASA?

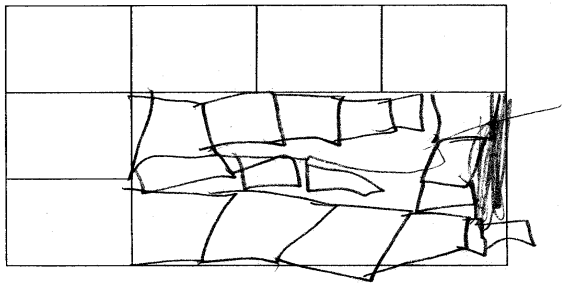
PASA is an innovative way of investigating young children's understanding of mathematics from Foundation to Year 2.

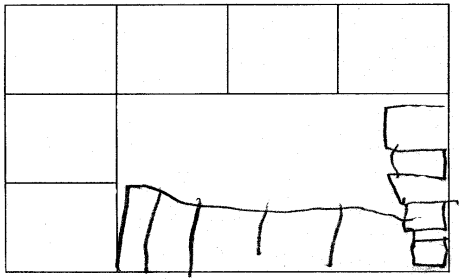
PASA seeks to find out how children think about mathematical ideas underlying tasks, rather than focusing on what mathematics children can and cannot do. By knowing how children approach tasks, teachers can plan and scaffold individual learning experiences.

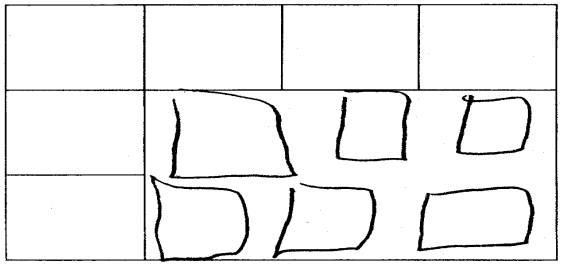
Stages of Structural Development

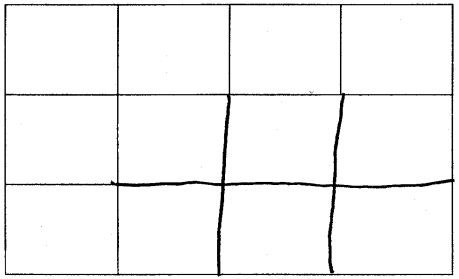
5 **Advanced structural** Response shows an accurate, efficient and generalised use of the underlying
structure
4 **Structural Response** shows a correct but limited use of the underlying structure
3 **Partial structural** Response shows most of the relevant features of the pattern, but the
underlying structural organisation is inaccurate or incomplete
2 **Emergent Response** shows some relevant features of the pattern, but these are not
organised in such a way as to reflect the underlying structure
1 **Prestructural** If any response is given, it shows only limited and disconnected features of the Pattern.

Development of structure









Key features

TASKS ACROSS CONCEPTS (CURRIULUMSTRANDS)

PASA consists of three one-on-one assessments. Each assessment consists of approximately 15 tasks, covering a wide variety of ideas that underlie strands in the early years Australian mathematics curriculum

The assessment provides information on the strengths and weaknesses of children at either end of the ability range

Components

RESPONSE BOOKLETS

- Response booklet for each PASA— Foundation, 1 and 2 that completed by teacher and child at interview
- Children write/draw their responses in the booklet
- Teachers categorise and record responses for one of five *Structural* levels

Components

TEACHER GUIDE

- A *Teacher Guide* with detailed information on all stages of the PASA assessment.
- An *Assessment Materials Kit* that contains a set of reusable stimulus materials for use when administering PASA.
- *Using PASA with PAT Maths*, a short guide that provides information for teachers wishing to use PASA with PAT Maths

PASA 2 Items

1. Partitioning Length into Thirds
2. Border Pattern
3. Triangular Array
4. Partitioning Money
5. Ten Frames
6. Skip Counting by Threes
7. Spatial Pattern Continuation
8. Square Array
9. Hundred Chart
10. The Clock
11. Grid Completion
12. Comparing Triangles
13. Growing Pattern Continuation
14. Making a Ruler
15. Bar Chart
16. Comparing Capacities

Five PASA groupings of tasks (structures)

1 Sequences
PASAF: 1, 2, 7 PASA1: 1, 2, 7 PASA2: 2, 7, 13

2 Shape and alignment
PASAF:2, 3, 8, 11, 14 PASA1: 3, 7, 8, 11 PASA2: 3, 7, 8, 11

3 Equal spacing
PASAF: 4, 5, 12, 13 PASA1: 10, 14 PASA2: 10, 14, 15

4 Structured Counting
PASAF: 3, 6, 8, 9, 10 PASA1: 3, 6, 8, 9, 13 PASA2: 3, 5, 6, 8, 9, 13

5 Partitioning
PASAF:4, 5, 11, 13, 14, 15 PASA1: 4, 5, 11, 12 PASA2:1, 4, 11, 12, 14, 16

5. Partitioning Length into Thirds

See Teacher Guide, p. 58

Materials: 30-cm strip of paper (flat, with no folds or marks)
pencil

Show the child the strip of paper and lay it flat on the table. Say:

Here is another strip of paper.

This time, I want to cut it into **three** pieces **exactly** the same size.

Indicate scissors cutting in the air with two fingers and ask:

Can you show me where I should cut it?

If in any doubt as to where the child is cutting, ask them to mark the cut(s) with a pencil.

Response Categories

5 **Advanced structural: Uses a partitioning procedure that ensures the three parts are equal**
- Successfully folds the strip into three equal parts

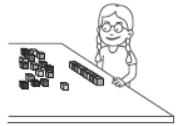
4 **Structural: Divides the length into three approximately equal parts and checks**
- Points to two places that divide the strip into three approximately equal parts, and checks that they are equal by some measurement process (e.g. using fingers)
- Attempts to fold the strip into three equal parts, but is unsuccessful

3 **Partial structural: Divides the length into three approximately equal parts**
- Points to two places that divide the strip into three approximately equal parts, using only a visual estimate

2 **Emergent: Divides the length into three unequal parts**
- Indicates two places that do not divide the strip into three approximately equal parts

1 **Prestructural: Does not divide the length into three parts**
- Makes no response
- Indicates one place or more than two places on the strip

1. Repeating Pattern



See Teacher Guide, p. 69

Materials: 1 green–blue–orange line of connected blocks
16 loose blocks: 4 green, 4 blue, 4 orange and 4 purple

Place the assembled line of blocks on the table in front of the child, with the orange block on the child's right. In addition, randomly arrange the loose blocks on the table. Say:

Look at this line of blocks.

I would like you to make another line **exactly** the same as this one.
You can use any of these blocks.

Observe carefully how the child creates the pattern, noticing whether they copy the line without hesitation or whether they hesitate after each block and frequently check that their copy matches the line.

If the child makes a line of a different length, point out that the copy must be exactly the same length as the original line and allow the child to make another attempt.

If the child makes an incorrect copy (Response Category 1), go on to Task 2.

If the child makes a correct copy, take away the original line of connected blocks and the extra blocks and ask:

Suppose I made the line longer here …

Point to the orange end of the child's line.

… keeping the same pattern, what would come next?

If the child only says the colour of the next block, ask for the colours of the following two blocks as well.

Response Categories

5 **Advanced structural: Uses the unit of repeat to copy and extend the pattern**
- Makes a correct copy, without frequent checking, and describes the correct extension as a group of three blocks

4 **Structural: Uses the unit of repeat to copy the pattern but not to extend it**
- Makes a correct copy, without frequent checking, and describes the correct extension one block at a time, rather than as a group of blocks

3 **Partial structural: Copies and extends the pattern without using the unit of repeat**
- Makes a correct copy, frequently checking, and describes the correct extension

2 **Emergent: Makes a correct copy, but an incorrect extension**
- Makes a correct copy, but describes an incorrect extension

1 **Prestructural: Makes an incorrect copy**
- Makes no response
- Makes an incorrect copy

Scoring and reporting

MANUAL SCORING OR ACER ANALYSIS

PASA can be *hand scored* by the teacher using the Teacher Guide and a downloadable proforma spreadsheet.

Spreadsheet creates individual and group diagnostic reports.

PASA scored automatically by using *ACER Test Scoring and Analysis*.
(Response booklets are sent to ACER with an order form indicating which reports are required)

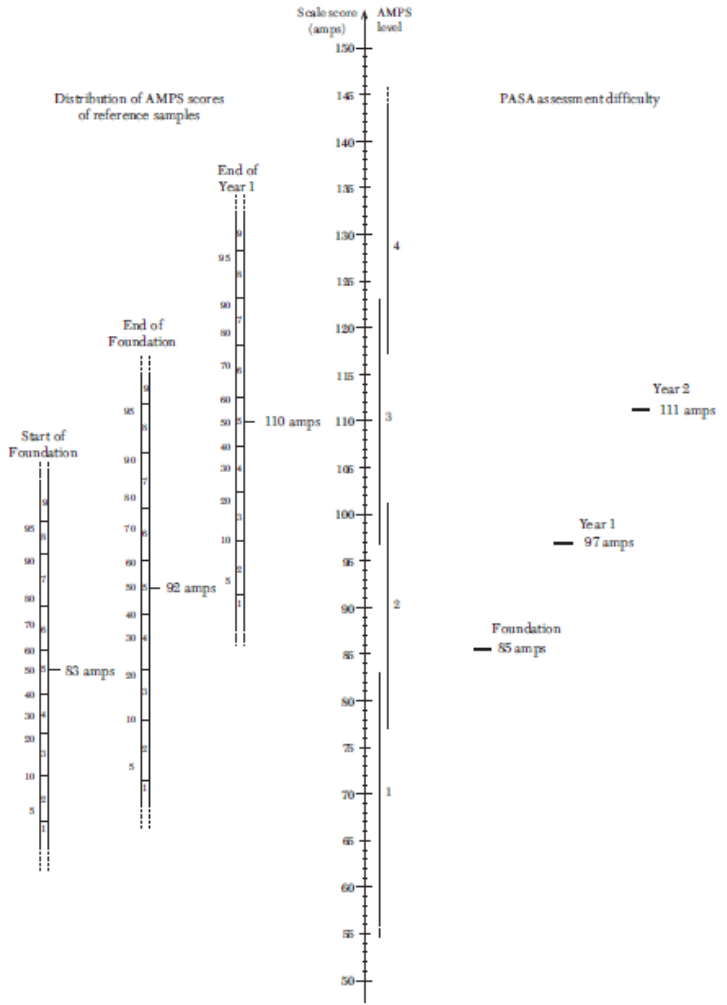
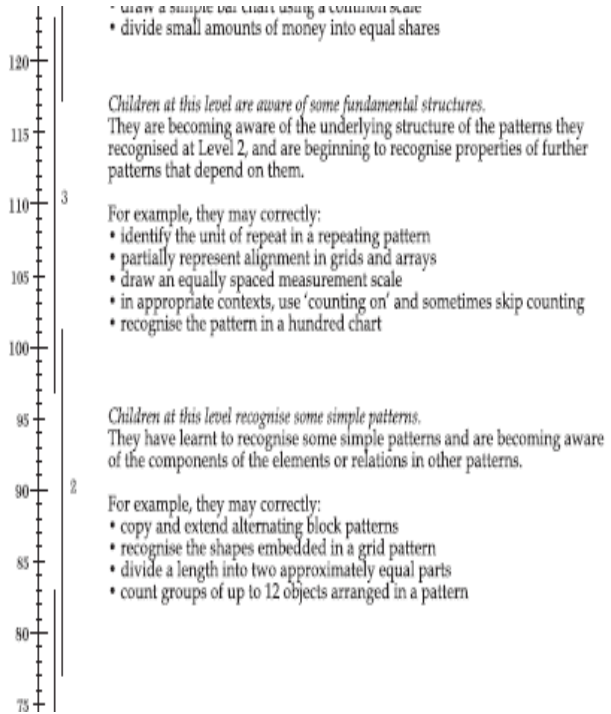


Figure 4.3: Distribution of scale scores of reference samples and difficulty of PASA assessments

PASA and curriculum outcomes



PASA assesses key concepts (across strands) central to mathematics curricula (see connection to Australian Curriculum-Mathematics)

PASA also assesses spatial visualisation, visual memory and representation

Responses shown as continuum of development (descriptors on the scale)

Pattern and Structure Mathematical Awareness Program (PASMAP)

Research-based challenging tasks

Parts 1 and 2 (Foundation- Year 1; Year 1-Year 3)

PASMAP model of learning on *structuring*

Pathways with Learning experiences linked to central concepts

Comprehensive teaching guide with examples of children's responses

Flexibility of use: integrate with existing programs to complete implementation use

PASMAP Pedagogical Approach

Modelling

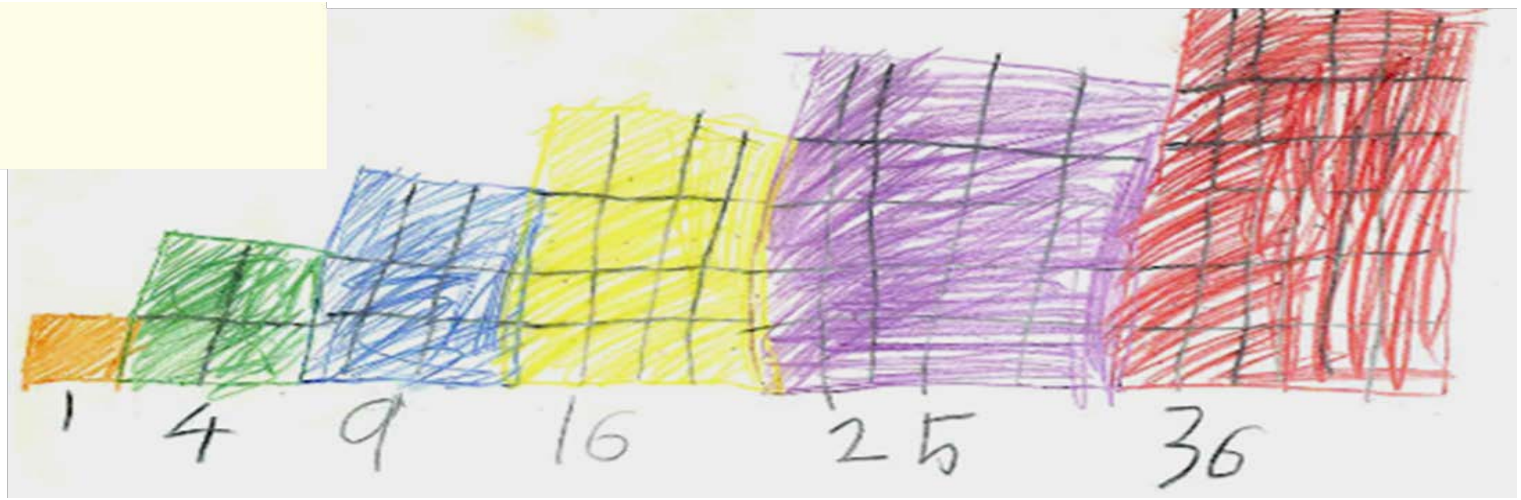
Representing

Visualising

Generalising

Sustaining

Pattern of Squares: Grid Structure

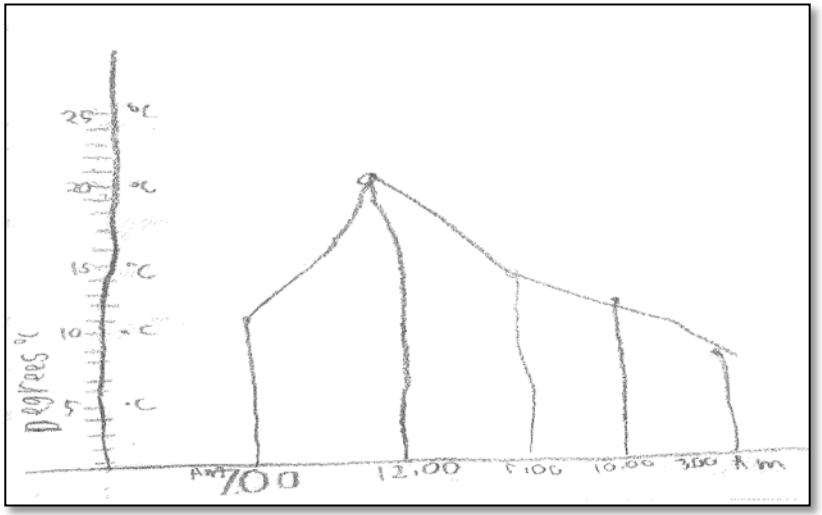


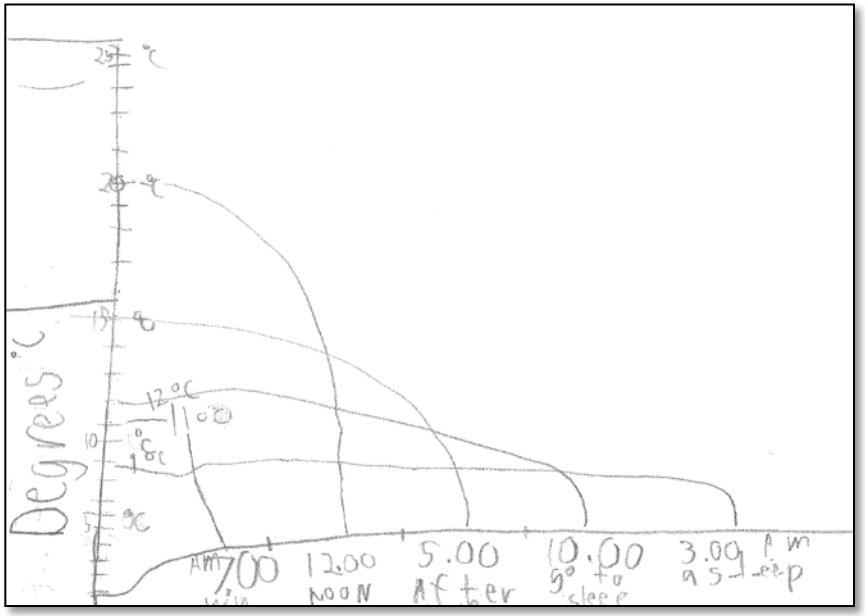
Foundation Level student 5.5 years: drawn freehand from memory following PASMAP implementation " it doesn't matter how big it gets they are always squares—one bigger across and down"

Where to from here?

- Monitoring of impact of PASA implementation on student learning outcomes and professional learning
- Long term outcomes: tracking individuals over time
- Upscaling— more diverse and larger samples of students across systems or contexts
- Investigate structural development across science eg data investigation
- Integrate impact of digital technologies on structural development
- Inform new analyses i.e. network analysis of PASA (Woolcott & Mulligan, 2105)

Structuring graphs: integrating horizontal and vertical axes





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