Case (Person) Fit and Residuals

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ConQuest 3 provides a variety of different opportunities to explore the compatibility of the data with the model that is being estimated. First, the fit of alternative models can be examined through comparisons of the deviance statistic. Second, residual-based fit statistics based upon the weighted (infit) and unweighted (outfit) of Wright and Masters (1982) are available for all item parameters. Third, generalised fit statistics of Adams and Wu (2009) which can be designed target specific alternative hypothesis of multidimensionality, local dependence and the like. Fourth, ability-group based fit statistics that use chi-squares to compare the performances of students in ability groups with their expected performance. Both generalised statistics and the ability group-based statistics can be computed for subgroups of cases. Finally ConQuest 3 can provide case fit statistics (person fit) and case-item residuals.

This note is concerned with the ConQuest 3 case fit statistics and case-item residuals.

Obtaining the Residuals and Case Fit Indices

To obtain residuals for every case and generalised item combination, use the `show` command with `residual` as argument. Additionally the option `estimate=wle` must be used and the output must be redirected to a file. Note that while output to a text file is the default the option `filetype=spss`, or `filetype=excel` can be used to direct the output to SPSS and EXCEL files respectively. If a case has not responded to a generalised item then the value –99 will reported as the residual for that case and generalised item combination.

The residual output file includes one record for each case. The first field of each record is a case sequence number, the second field is the case `pid` (if one was specified in the `format`) and the subsequent fields are the residuals for each generalised item.

To obtain case fit indices use the `show` command with case as argument and include the option `pfit=yes`. Additionally the option `estimate=wle` must be used and the output must be redirected to a file. Note that while output to a text file is the default the option `filetype=spss`, or `filetype=excel` can be used to direct the output to SPSS and EXCEL files, respectively.

The weighted likelihood estimate output file with `pfit=yes` includes one record for each case. The first field of each record is a case sequence number; the second field is the case `pid` (if one was specified in the `format`). These two fields are followed by the case score, maximum possible score, weighted likelihood estimate, standard error of the weighted likelihood estimate and the case fit. While just a single case fit statistic is provided, the multiple scores and estimates will be provided in the case of multidimensional models.

Models for which the Residuals and Case Fit Indices are available

The residuals and case fit indices are available for all ConQuest models and item parameter estimation methods. As both the residuals and case fit values are computed using weighted likelihood case estimates they will not be available for cases that do not have weighted likelihood estimates. Most notably, in the case of multidimensional models, a case must have responded to at least one item on each dimension.

Formulae for Case Fit and Residuals

If we let we let x_{ni} be the response of case n to generalised item i, then we can define the following expectation and variance:

$$E(x_{ni}) = \sum_{j=1}^{K_i} \frac{b_{ij} exp(b_{ij}\hat{\theta}_n + a'_{ij}\hat{\xi})}{\sum_{k=1}^{K_i} exp(b_{ik}\hat{\theta}_n + a'_{ik}\hat{\xi})}$$

and

$$var(x_{ni}) = \sum_{j=1}^{K_i} \frac{b_{ij}^2 exp(b_{ij}\hat{\theta}_n + a'_{ij}\hat{\xi})}{\sum_{k=1}^{K_i} exp(b_{ik}\hat{\theta}_n + a'_{ik}\hat{\xi})} - E^2(x_{ni})$$

where $\hat{\theta}_n$ is the weighted likelihood estimate for case n and $\hat{\xi}$ are the item parameter estimates. The b_{ij} and a'_{ij} values are elements of the scoring vectors and design matrices respectively. Appropriate choices of these values lead to specific models (see Adams and Wu (2007) for a full discussion). For Rasch models the b_{ij} values are fixed constants, whereas under models with free scores they are also estimated parameters. Further, note that the formulae provided above apply for the unidimensional model, they can be readily generalised for use with multidimensional models.

Having defined the expectation and variance we can define the standardised residual z_{ni} as

$$z_{ni} = \frac{x_{ni} - E(x_{ni})}{\sqrt{var(x_{ni})}}$$

and a case (out)fit index as:

$$pfit_n = \frac{1}{|\Omega_n|} \sum_{i \in \Omega_n} z_{ni}^2$$

Where Ω_n is the set of items that case n, responded to.

References

Adams, R. J., & Wu, M. L. (2007). The mixed-coefficient multinomial logit model: A generalized form of the Rasch model. In M. v. Davier & C. H. Carstensen (Eds.), *Multivariate and mixture distribution Rasch models: Extensions and applications* (pp. 57 – 76): Springer Verlag.

Adams, R.J. & Wu, M.L. (2009) The Construction and Implementation of User-Defined Fit Tests for Use with Marginal Maximum Likelihood Estimation and Generalized Item Response Models *Journal of Applied Measurement, 10*(4), 355-370.

Wright, B.D., and Masters, G.N. 1982. Rating Scale Analysis: Rasch Measurement. Chicago: MESA Press.