Multidimensional Models

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ConQuest analyses are not restricted to models that involve a single latent dimension. ConQuest can be used for the analysis of sets of items that are designed to produce measures on up to 30 latent dimensions[1]. In this tutorial, multidimensional models are fitted to data that were analysed in previous tutorials using a one-dimensional model. In doing so, we are able to use ConQuest to explicitly test the unidimensionality assumption made in the previous analyses. We are also able to illustrate the difference between derived estimates and ConQuest's direct estimates of the correlation between latent variables. In this tutorial, we also introduce the two different approaches to estimation (quadrature and Monte Carlo) that ConQuest offers; and in the latter part of the tutorial, we discuss and illustrate two types of multidimensional tests: multidimensional between-item and multidimensional within-item tests.

FITTING A TWO-DIMENSIONAL MODEL

In the first sample analysis in this tutorial, the data used in tutorial one is re-analysed. In that tutorial, we described a data set that contained the responses of 1000 students to 12 multiple choice items, and the data were analysed as if they were from a unidimensional set of items. This was a bold assumption, because these data are actually the responses of 1000 students to six mathematics multiple choice items and six science multiple choice items.

The files used in this sample analysis are:

`ex7a.cqc`	The command statements.
`ex1.dat`	The data.
`ex1.lab`	The variable labels for the items on the multiple choice test.
`ex7a.shw`	The results of the Rasch analysis.
`ex7a.itn`	The results of the traditional item analyses.
`ex7a.eap`	The EAP ability estimates for the students.
`ex7a.mle`	The maximum likelihood ability estimates for the students.

The contents of the command file `ex7a.cqc` are shown in Figure 1.

1 Although ConQuest will permit the analysis of up to 30 dimensions, our simulation studies suggest that there may be moderate bias in the estimates of the latent covariance matrix for models with more than eight dimensions (Volodin and Adams, 1995).

```
1.  datafile ex1.dat;
2.  format id 1-5 responses 12-23;
3.  labels << ex1.lab;
4.  key acddbcebbacc ! 1;
5.  score (0,1) (0,1) ( ) !items(1-6);
6.  score (0,1) ( ) (0,1) !items(7-12);
7.  model item;
8.  estimate;
9.  show !estimates=latent,tables=1:2:3:4 >> ex7a.shw;
10. itanal >> ex7a.itn;
11. show cases !estimates=eap >> ex7a.eap;
12. show cases !estimates=mle >> ex7a.mle;
```

Figure 1 Command File for a Two-dimensional Dichotomous Test

1. Indicates the name and location of the data file. Any name that is valid for the computer you are using can be used here.

2. The `format` statement describes the layout of the data in the file `ex1.dat`.

3. Reads a set of item labels from the file `ex1.lab`.

4. Recodes the correct responses to 1 and all other values to 0.

5.-6. The fact that a multidimensional model is to be fitted is indicated by the `score` statement syntax. In our previous uses of the `score` statement, the argument has had two lists, each in parentheses—a *from* list and a *to* list. The effect of those `score` statements was to assign the scores in the *to* list to the matching codes in the *from* list. If a multidimensional model is required, additional *to* lists are added. The arguments of the two `score` statements here each contain three lists. The first is the *from* list and the next two are *to* lists, one for each of two dimensions. The first six items are scored on dimension one; hence, the second *to* list in the first `score` statement is empty. The second six items are scored on the second dimension; hence, the first *to* list in the second `score` statement is empty.

7. The simple logistic model is used.

8. The model will be estimated using default settings.

Note:	***The default settings will result in a Gauss-Hermite method that uses15 nodes for each latent dimension when performing the integrations that are necessary in the estimation algorithm. For a two-dimensional model, this means a total of 15×15=225 nodes. The total number of nodes that will be used increases exponentially with the number of dimensions, and the amount of time taken per iteration increases linearly with the number of nodes. In practice, we have found that 5000 nodes is a reasonable upper limit on the total number of nodes that can be used.***

9. This `show` statement writes tables 1, 2, 3, and 4 into the file `ex7a.shw`. Displays of the ability distribution will represent the distribution of the latent variable.

10. The `itanal` statement writes item statistics to the file `ex7a.itn`.

11. This `show` statement writes a file containing EAP ability estimates for the students on both estimated dimensions.

12. This `show` statement writes a file containing maximum likelihood ability estimates for the students on both estimated dimensions.

RUNNING THE TWO-DIMENSIONAL SAMPLE ANALYSIS

To run this sample analysis, start the gui version of ConQuest and open the control file

`Ex7a.cqc`

Select Run -> Run All. ConQuest will begin executing the statements that are in the file ex7a.cqc; and as they are executed, they will be echoed in the Output Window. When ConQuest reaches the `estimate` statement, it will begin fitting a multidimensional form of Rasch's simple logistic model to the data. As it does so, it will report on the progress of the estimation. This particular sample analysis will take 140 iterations to converge. Figure 2 is a sample of the information that will be reported by ConQuest as it iterates to find the parameter estimates.

This is the current estimate of the covariance matrix expressed as a correlation matrix.

the covariance matrix for the two dimensions.

```
Iteration: 91
Deviance =    13244.77825

Covariance Estimates:
Dimension 1     1.24889   0.78826
Dimension 2     0.78826   0.78800

Correlation Estimates:
Dimension 1     1.00000   0.79459
Dimension 2     0.79459   1.00000


Means:
Dimension 1     0.80039
Dimension 2     1.36494

Maximum changes:
Item parameter estimates ==>    0.00003 (Parameter 8)
Mean estimates           ==>   -0.00007 (Dimension 2)
Covariance matrix        ==>   -0.00032 (Element 2  2)
Change in the deviance is       0.00261
```

These are the current estimates of the means for the two dimensions.

Figure 2 Reported Information on Estimation Progress for `ex7a.cqc`

In Figure 3, we have reported the first table (table 1) from the file `ex7a.shw`. From this figure, we note that the multidimensional model has estimated 15 parameters; they are made up of 10 item difficulty parameters, the means of the two latent dimensions, and the three unique elements of the variance-covariance matrix. Ten item parameters are used to describe the 12 items because identification constraints are applied to the last item on each dimension. The deviance for this model is 13244.73. If we refer back to tutorial one, we note that a unidimensional model when fitted to these data required the estimation of 13 parameters —11 item difficulty parameters, one mean, and one variance—and the deviance was 13274.88. As the unidimensional model is a submodel of the two-dimensional model, the difference between the deviance of these two models is distributed as a chi-square with two degrees of freedom. Given the estimated difference of 30.1 in the deviance, we conclude that the unidimensional model does not fit these data as well as the two-dimensional model does.

```
===============================================================================
ConQuest: Generalised Item Response Modelling Software     Tue Oct 31 16:05 2006
SUMMARY OF THE ESTIMATION
===============================================================================

Estimation method was: Gauss-Hermite Quadrature with 225 nodes
Assumed population distribution was: Gaussian
Constraint was: DEFAULT
The Data File: ex1.dat
The format:  id 1-5 responses 12-23
The regression model:
Grouping Variables:
The item model: item
Sample size: 1000
Final Deviance:    13244.72950
Total number of estimated parameters: 15
The number of iterations: 140
Termination criteria:  Max iterations=1000, Parameter Change= 0.00010
                       Deviance Change= 0.00010
Iterations terminated because the convergence criteria were reached
Random number generation seed:    1.00000
Number of nodes used when drawing PVs: 2000
Number of nodes used when computing fit: 200
Number of plausible values to draw: 5
Maximum number of iterations without a deviance improvement: 100
Maximum number of Newton steps in M-step: 10
Value for obtaining finite MLEs for zero/perfects:    0.30000
===============================================================================
```

The deviance for this model is 30.1 less than that for the one-dimensional model that was fitted to the same data in Chapter 3. This value follows a chi-square distribution with two degrees of freedom. It follows that we would reject the hypothesis that the undimensional model fits these data as well as the two-dimensional model.

Figure 3 Summary Information for the Two-dimensional Model

Figure 4 shows the second table (table 2) that is produced by the first `show` statement. It contains the item difficulty estimates and the fit statistics. It is interesting to note that the fit statistics reported here are almost identical to those reported for the unidimensional model. Note also that two of the item parameters are constrained. For identification purposes, the mean of the item parameters on each dimension is constrained to be zero. This is achieved by choosing the difficulty of the last item on each dimension to be equal to the negative sum of the difficulties of the other items on the dimension. As an alternative approach, it is possible to use the `constraints` argument of the `set` command to force the means of the latent variables to be set at zero and to allow all item parameters to be free.

```
==============================================================================
ConQuest: Generalised Item Response Modelling Software     Tue Oct 31 16:05 2006
TABLES OF RESPONSE MODEL PARAMETER ESTIMATES
==========================================================================
TERM 1: item
--------------------------------------------------------------------------------
   VARIABLES                        UNWEIGHTED FIT            WEIGHTED FIT
-------------                    -----------------------  -----------------------
     item     ESTIMATE  ERROR^   MNSQ       CI        T    MNSQ       CI        T
--------------------------------------------------------------------------------
 1   BSMMA01    0.056   0.055    0.87 ( 0.91, 1.09) -3.0   0.91 ( 0.94, 1.06) -2.8
 2   BSMMA02   -0.515   0.057    1.10 ( 0.91, 1.09)  2.2   1.02 ( 0.92, 1.08)  0.5
 3   BSMMA03   -0.354   0.056    0.91 ( 0.91, 1.09)                              6
 4   BSMMA04    0.555   0.054    0.99 ( 0.91, 1.09)                              8
 5   BSMMA05    0.917   0.054    1.17 ( 0.91, 1.09)                              9
 6   BSMMA06   -0.659*  0.123    1.00 ( 0.91, 1.09)                              2
 7   BSMSA07   -0.079   0.052    1.04 ( 0.91, 1.09)                              2
 8   BSMSA08   -0.014   0.052    1.10 ( 0.91, 1.09)  2.2   1.06 ( 0.92, 1.08)  1.5
 9   BSMSA09   -0.648   0.056    0.91 ( 0.91, 1.09) -2.0   0.97 ( 0.88, 1.12) -0.6
 10  BSMSA10   -0.079   0.052    1.08 ( 0.91, 1.09)  1.8   1.03 ( 0.92, 1.08)  0.7
 11  BSMSA11   -0.186   0.053    0.91 ( 0.91, 1.09) -2.2   0.96 ( 0.91, 1.09) -0.9
 12  BSMSA12    1.005*  0.119    0.99 ( 0.91, 1.09) -0.2   0.99 ( 0.95, 1.05) -0.3
--------------------------------------------------------------------------
An asterisk next to a parameter estimate indicates that it is constrained
Separation Reliability =  0.987
Chi-square test of parameter equality =     673.95,  df = 10,  Sig Level = 0.000
^ Quick standard errors have been used
==============================================================================
```

Constraints are applied to make the mean of the item difficulties on each dimension zero.

Figure 4 Item Parameter Estimates for the Two-dimensional Model

Figure 5 shows the estimates of the population parameters as they appear in the third table (table 3) in file `ex7a.shw`. The first panel of the table shows that the estimated mathematics mean is 0.800 and the estimated science mean is 1.363.

NOTE:	*This does not mean that this sample of students is more able in science than in mathematics. The origin of the two scales has been set by making the mean of the item difficulty parameters on each dimension zero, and no constraints have been placed upon the variances. Thus, these are two separate dimensions; they do not have a common unit or origin.*

The second panel of the table shows the variances, covariance and correlation for these two dimensions. The correlation between the mathematics and science latent variables is 0.802. Note that this correlation is effectively corrected for any attenuation caused by measurement error.

```
===============================================================================
ConQuest: Generalised Item Response Modelling Software     Tue Oct 31 16:05 2006
TABLES OF POPULATION MODEL PARAMETER ESTIMATES
===============================================================================
REGRESSION COEFFICIENTS

                                     Dimension
                         ---------------------------
Regression Variable         Dimension 1      Dimension 2

CONSTANT                  0.800 ( 0.035)    1.363 ( 0.028)
----------------------------------------------------------------
An asterisk next to a parameter estimate indicates that it is constrained
================================================================
COVARIANCE/CORRELATION MATRIX

                            Dimension
                         ----------------
                            1        2

 Dimension 1                       0.790
 Dimension 2              0.802
--------------------------------------------
Variance                  1.245    0.779
--------------------------------------------
An asterisk next to a parameter estimate indicates that it is constrained
Values below the diagonal are correlations and values above are covariances
===============================================================================
```

These are the estimated means for the two dimensions.

This is the estimated correlation between the dimensions.

This is the estimated covariance between the dimensions.

These are the estimated variances for the two dimensions.

Figure 5 Population Parameter Estimates for the Two-dimensional Model

Figure 6 is the last table (table 4) from the file `ex7a.shw`. The left panel shows a representation of the latent mathematics ability distribution, and the right panel indicates the difficulty of the mathematics items. In the unidimensional equivalent of this figure, the items are plotted so that a student with a latent ability estimate that corresponded to the level at which the item was plotted would have a 50% chance of success on that item. For the multidimensional case, each item is assigned to a single dimension. A student whose latent ability estimate on that dimension is equal to the difficulty estimate for the item would have a 50% chance of success on that item.

```
================================================================================
ConQuest: Generalised Item Response Modelling Software     Tue Oct 31 16:05 2006
MAP OF LATENT DISTRIBUTIONS AND RESPONSE MODEL PARAMETER ESTIMATES
================================================================================
        Dimension        Terms in the Model (excl Step terms)
--------------------
Dimension 1                               +item
-------------------------------------------------------------------
                          |                                       |
                         X|                                       |
                         X|                                       |
                         X|                                       |
                        XX|                                       |
   3                     X|                                       |
                       XXX|                                       |
                      XXXX|                                       |
                     XXXXX|                                       |
                    XXXXXX|                                       |
                  XXXXXXXX|                                       |
   2              XXXXXXXX|                                       |
                 XXXXXXXXX|                                       |
             XXXXXXXXXXXXX|                                       |
                XXXXXXXXXX|                                       |
            XXXXXXXXXXXXXX|                                       |
              XXXXXXXXXXXX|                                       |
             XXXXXXXXXXXXX|                                       |
   1        XXXXXXXXXXXXXX|                                       |
           XXXXXXXXXXXXXXX|5                                      |
              XXXXXXXXXXXX|                                       |
         XXXXXXXXXXXXXXXXX|4                                      |
         XXXXXXXXXXXXXXXXX|                                       |
           XXXXXXXXXXXXXXX|                                       |
               XXXXXXXXXXX|                                       |
   0          XXXXXXXXXXXX|1                                      |
               XXXXXXXXXXX|                                       |
                 XXXXXXXXX|3                                      |
                    XXXXXX|2                                      |
                  XXXXXXXX|6                                      |
                     XXXXX|                                       |
                     XXXXX|                                       |
  -1                   XXX|                                       |
                       XXX|                                       |
                       XXX|                                       |
                        XX|                                       |
                        XX|                                       |
                         X|                                       |
  -2                      |                                       |
===================================================================
Each 'X' represents   3.5 cases

===================================================================
```

This is the estimated latent mathematics ability distribution.

These are the mathematics items plotted at their difficulty estimates.

Figure 6 Map of the Latent Variables for the Two-dimensional Model

EXTENSION:	*If quadrature-based estimation is used, the computation time needed to fit multidimensional models increases rapidly as additional dimensions are added. This can be alleviated somewhat by reducing the number of nodes being used, although reducing the number of nodes by too much will affect the accuracy of the parameter estimates. With this particular sample analysis, the use of 10 nodes per dimension results in variance estimates that are greater than those obtained using 20 nodes per dimension and the deviance is somewhat higher. If 30 nodes per dimension are used, the results are equivalent to those obtained with 20 nodes.* *If you want to explore the possibility of using quadrature with less than 20 nodes per dimension, then we recommend fitting the model with a smaller number of nodes (e.g., 10) and then gradually increasing the number of nodes, noting the impact that the increased number of nodes has on parameter estimates, most importantly the variance. When you reach a point where increasing the number of nodes does not change the parameter estimates, including the variance, then you can have some confidence that an appropriate number of nodes has been chosen.*

COMPARING THE LATENT CORRELATION WITH OTHER CORRELATION ESTIMATES

The last two `show` statements in Figure 1 produced files of students' EAP and maximum likelihood ability estimates respectively. From these files we are able to compute the product moment correlations between the various ability estimates. In a run not reported here, we also fitted separate unidimensional models to the mathematics and science items and from those analyses produced EAP ability estimates. The various correlations that can be computed between mathematics and science are reported in Table 9-1.[2]

Table 9-1 Comparison of Some Correlation Estimates with the Latent Ability Estimates

Analysis Method	*Correlation*
Estimates*	
Raw Scores	0.396
Multidimensional EAP	0.933
Unidimensional EAP	0.399
MLE	0.394
Direct ConQuest Estimation	**0.774**

* These are ability estimates computed by ConQuest and then used to estimate the correlation.

The estimates based on the raw score, unidimensional EAP, and MLE, which are all similar, indicate a correlation of about 0.40 between mathematics and science. All three estimates are attenuated substantially by measurement error. As the estimated KR-20 reliability of each of

2 The file `ex7a.out` (provided with the samples) contains the data used in computing the results shown in Table 9-1. The fixed-format file contains eight fields in this order: mathematics raw score, science raw score, mathematics MLE, science MLE, mathematics EAP from the joint calibration, science EAP from the joint calibration, mathematics EAP from separate calibrations, and science EAP from separate calibrations.

these dimensions is 0.58 and 0.43 respectively, an application of the standard 'correction for attenuation' formula yields estimated correlations of about 0.80.[3] This value is in fairly close agreement with the ConQuest estimate. The correlation of 0.933 between the EAP estimates derived from the two-dimensional analysis is a dramatic overestimation of the correlation between these two variables and should not be used. This overestimation occurs because the EAP estimates are 'shrunken' towards each other. The degree of shrinkage is a function of the reliability of measurement on the individual dimensions; so if many items are used for each dimension, then all of the above indices will be in agreement.

> *EXTENSION:* ***It is possible to recover the ConQuest estimate of the latent ability correlation from the output of a multidimensional analysis by using plausible values instead of EAP estimates. Plausible values can be produced through the use of the `cases` argument and the `estimates=latent` option of the `show` command.***

HIGHER-DIMENSIONAL ITEM RESPONSE MODELS

ConQuest can be used to fit models of up to 15 dimensions, and we have routinely used it with up to six dimensions. When analysing data with three or more dimensions, a Monte Carlo approach to the calculation of the integrals should be used. In this sample analysis, we fit a five-dimensional model to some performance assessment data that were collected in Australia as part of the TIMSS study (Lokan, Ford and Greenwood, 1996). The data consist of the responses of 583 students to 28 items that belong to five different performance assessment tasks. These data are quite sparse because each student was only required to undertake a small subset of the tasks, but every task appears at least once with every other task.

The files that will be used in this sample analysis are:

`ex7b.cqc`	The command statements that we use.
`ex7b.dat`	The data.
`ex7b.lab`	The variable labels for the items.
`ex7b.prm`	The estimates of the item response model parameters.
`ex7b.reg`	The estimates of the regression coefficients for the population model.
`ex7b.cov`	The estimates of the variance-covariance matrix for the population model.
`ex7b.shw`	The results of the Rasch analysis.

The contents of file `ex7b.cqc` are shown in Figure 7.

[3] Here we are using the KR-20 index that is reported by ConQuest at the end of the printout from an `itanal` analysis.

```
1.  title Australian Performance Assessment Data;
2.  datafile ex7b.dat;
3.  format    responses 1-28;
4.  codes     0,1,2,3;
5.  labels << ex7b.lab;
6.  recode (2) (1) !items(9,10);
7.  recode (3) (2) !items(25);
8.  score (0,1,2,3) (0,1,2,3) ( ) ( ) ( ) ( ) ! items (1-6);
9.  score (0,1,2,3) ( ) (0,1,2,3) ( ) ( ) ( ) ! items (7-13);
10. score (0,1,2,3) ( ) ( ) (0,1,2,3) ( ) ( ) ! items (14-17);
11. score (0,1,2,3) ( ) ( ) ( ) (0,1,2,3) ( ) ! items (18-25);
12. score (0,1,2,3) ( ) ( ) ( ) ( ) (0,1,2,3) ! items (26-28);
13. model item+item*step;
14. set warnings=no,update=yes;
15. export parameters >> ex7b.prm;
16. export reg_coefficients >> ex7b.reg;
17. export covariance >> ex7b.cov;
18. import init_parameters <<ex7b.prm;
19. import init_reg_coefficents <<ex7b.reg;
20. import init_covariance << ex7b.cov;
21. estimate!method=montecarlo,nodes=2000,converge=.005;
22. show !tables=1:2:3:4,estimates=latent >>ex7b.shw;
23. quit;
```

Figure 7 Command File for a Higher-dimensional Item Response Model

1. Gives the title.
2. Gives the name of the data file to be analysed. In this case, the data are contained in the file `ex7b.dat`.
3. The `format` statement indicates that there are 28 items, and they are in the first 28 columns of the data file.
4. Restricts the valid codes to 0, 1, 2 or 3.
5. A set of labels for the items are to be read from the file `ex7b.lab`.

6.-7. If a gap occurs in the scores in the response data for an item, then the next higher score for that item must be recoded downwards to close the gap. For example, in this data set, by coincidence, no response to item 9 or item 10 was scored as 1; all responses to these two items were scored as 0 or 2. To fill the gap between 0 and 2, the 2 has been recoded to 1 by the first `recode` statement. Similarly, for item 25, none of the response data is equal to 2, so 3 must be recoded to 2 to fill the gap.

NOTE:	*The model being fitted here is a partial credit model. Therefore, all score categories between the highest category and the lowest category must contain data. If this is not the case, then some parameters will not be identified. If* **`warnings`** *is not set to* **`no`**, *then ConQuest will flag those parameters that are not identified and will indicate that recoding of the data is necessary. If* **`warnings`** *is set to* **`no`**, *then the parameters that are not identified due to null categories will not be reported. If a rating scale model were being fitted to these data, then recoding would not be necessary because all of the step parameters would be identified.*

> *EXTENSION:* ***Wilson and Masters (1993) discuss a method of dealing with data that have 'null' categories of the type we observe in these data for items 9, 10 and 25. Their approach can be implemented easily in ConQuest by using a `score` statement that assigns a score of 2 to the category 1 of items 9 and 10 and a score of 3 to the category 2 of item 25, after recoding has been done to close the gaps.***

8.-12. The model that we are fitting here is five dimensional, so the `score` statements contain six sets of parentheses as their arguments, one for the *from* codes and five for the *to* codes. The option of the first `score` statement gives the items to be assigned to the first dimension, the option of the second `score` statement gives the items to be allocated to the second dimension, and so on.

13. The model we are using is the partial credit model.

14. We want to update the export files of parameter estimates (see lines 15 through 17) every iteration, without warnings.

15.-17. Request that item, regression and covariance parameter estimates be written to the files `ex7b.prm`, `ex7b.reg`, and `ex7b.cov` respectively.

18.-20. Initial values of item, regression and covariance parameter estimates are to be read from the files `ex7b.prm`, `ex7b.reg`, and `ex7b.cov` respectively.

> *NOTE:* ***We have used the same names for the initial value and export files. These files must already exist so that, before the estimation commences, initial values can be read from them. After each iteration, the values in these files are then updated with the current parameter estimates. Importing and exporting doesn't happen until the `estimate` statement is executed; thus, the order of the `import` and `export` statements is irrelevant, so long as they precede the `estimate` statement.***

21. This `estimate` statement has three arguments: `method=montecarlo` requests that the integrals that are computed in the estimation be approximated using Monte Carlo methods; `nodes=2000` requests 2000 nodes be used in computing integrals; and `converge=.005` requests that the estimation be terminated when the largest change in any parameter estimate between successive iterations becomes less than 0.005.

RUNNING A HIGHER-DIMENSIONAL SAMPLE ANALYSIS

To run this sample analysis, launch the console version of ConQuest by typing the command

```
ConQuestCMD ex7b.cqc
```

ConQuest will begin executing the statements that are in the file `ex7b.cqc;` and as they are executed, they will be echoed on the screen. When ConQuest reaches the `estimate` statement, it will begin fitting a multidimensional form of Rasch's simple logistic model to the data. As it does so, it will report on the progress of the estimation. This particular sample analysis will take 30 iterations to converge.

Figures 8, 9 and 10 show three of the tables (2, 3 and 4) that are written to `ex7b.shw.`

In Figure 8, note that five items have their parameter estimates constrained. These are the five items that are listed as the last item on each of the dimensions. Their values are constrained to ensure that the mean of the item parameters for each dimension is zero.

```
============================================================================
Australian Performance Assessment Data                  Sun Feb 18 10:13 2007
TABLES OF RESPONSE MODEL PARAMETER ESTIMATES
============================================================================
TERM 1: item
```

	VARIABLES			UNWEIGHTED FIT			WEIGHTED FIT		
	item	ESTIMATE	ERROR^	MNSQ	CI	T	MNSQ	CI	T
1	bspm11	-1.126	0.132	1.65	(0.80, 1.20)	5.4	1.18	(0.47, 1.53)	0.7
2	bspm12	0.024	0.128	1.01	(0.80, 1.20)	0.2	0.93	(0.79, 1.21)	-0.6
3	bspm13	-0.579	0.121	0.90	(0.80, 1.20)	-1.0	0.90	(0.57, 1.43)	-0.5
4	bspm14	0.395	0.104	1.05	(0.80, 1.20)	0.5	1.01	(0.80, 1.20)	0.1
5	bspm15a	-0.836	0.138	0.73	(0.80, 1.20)	-2.8	0.86	(0.69, 1.31)	-0.9
6	bspm15b	2.122*	0.280	0.96	(0.80, 1.20)	-0.4	1.00	(0.86, 1.14)	-0.0
7	bspm21	-2.804	0.114	0.18	(0.80, 1.20)	-12.7	0.77	(0.30, 1.70)	-0.7
8	bspm22	0.702	0.080	1.02	(0.80, 1.20)	0.3	1.03	(0.83, 1.17)	0.3
9	bspm23	-1.426	0.105	0.70	(0.80, 1.20)	-3.2	0.84	(0.73, 1.27)	-1.2
10	bspm24	-0.301	0.099	0.93	(0.80, 1.20)	-0.7	0.91	(0.85, 1.15)	-1.2
11	bspm25	0.622	0.082	0.97	(0.80, 1.20)	-0.2	1.01	(0.83, 1.17)	0.1
12	bspm26a	1.580	0.074	1.15	(0.79, 1.21)	1.4	1.08	(0.81, 1.19)	0.9
13	bspm26b	1.628*	0.229	[illegible]	[illegible]	[illegible]	1.15	(0.80, 1.20)	1.4
14	bspm31	-0.034	0.074	[illegible]	[illegible]	[illegible]	1.04	(0.78, 1.22)	0.4
15	bspm32	-0.749	0.077	[illegible]	[illegible]	[illegible]	0.87	(0.73, 1.27)	-1.0
16	bspm33	-0.182	0.074	[illegible]	[illegible]	[illegible]	0.96	(0.77, 1.23)	-0.3
17	bspm34	0.965*	0.130	[illegible]	[illegible]	[illegible]	1.15	(0.79, 1.21)	1.3
18	bspm41	-1.181	0.109	1.11	(0.80, 1.20)	1.1	1.06	(0.75, 1.25)	0.5
19	bspm42	-0.698	0.093	0.79	(0.80, 1.20)	-2.1	0.95	(0.81, 1.19)	-0.5
20	bspm43	-0.516	0.104	1.43	(0.80, 1.20)	3.8	1.31	(0.83, 1.17)	3.4
21	bspm44	-1.205	0.110	1.27	(0.80, 1.20)	2.4	1.15	(0.75, 1.25)	1.2
22	bspm45a	0.221	0.069	0.82	(0.80, 1.20)	-1.8	0.87	(0.82, 1.18)	-1.4
23	bspm45b	0.356	0.111	0.98	(0.80, 1.20)	-0.2	0.98	(0.83, 1.17)	-0.2
24	bspm45c	-0.212	0.083	0.84	(0.80, 1.20)	-1.6	0.85	(0.82, 1.18)	-1.8
25	bspm46	3.235*	0.260	0.59	(0.80, 1.20)	-4.7	0.89	(0.39, 1.61)	-0.3
26	bspm51	-0.323	0.066	1.10	(0.80, 1.20)	1.0	1.07	(0.81, 1.19)	0.7
27	bspm52	-0.245	0.076	0.83	(0.80, 1.20)	-1.8	0.85	(0.83, 1.17)	-1.8
28	bspm53	0.568*	0.100	1.15	(0.80, 1.20)	1.5	1.09	(0.82, 1.18)	0.9

Five item parameters are constrained, one for each dimension.

Figure 8 Item Parameter Estimates for a Five-dimensional Sample Analysis

Figure 9 shows the population parameter estimates, which in this case consist of means for each of the dimensions and the five-by-five variance-covariance matrix of the latent dimensions.

```
========================================================
REGRESSION COEFFICIENTS

                                                 Dimensi
                   -------------------------------------------------------------------------
Regression Variable  Dimension 1    Dimension 2     Dimension 3   Dimension 4    Dimension 5

CONSTANT           1.677 ( 0.055) 0.543 ( 0.045) 1.164 ( 0.060) 0.112 ( 0.034) -0.028 ( 0.046)
--------------------------------------------------------------------------------------------
An asterisk next to a parameter estimate indicates that it is constrain
=======================================================================
COVARIANCE/CORRELATION MATRIX

                                             Dimension
                         --------------------------------------------
D                           1         2         3         4         5

 Dimension 1                       0.790     0.785     0.438     0.229
 Dimension 2              0.550              0.849     0.372     0.282
 Dimension 3              0.407    0.541               0.726     0.844
 Dimension 4              0.408    0.426     0.619               0.456
 Dimension 5              0.156    0.236     0.525     0.510
----------------------------------------------------------------------
Variance                  1.766    1.169     2.108     0.652     1.225
----------------------------------------------------------------------
```

There are now five means, one for each dimension.

These are the covariances between the dimensions.

These are the correlations between the dimensions.

There are now five variances, one for each dimension.

Figure 9 Population Model Parameter Estimates for the Five-dimensional Sample Analysis

> *EXTENSION:* *As an alternative to identifying the model by making the mean of the item parameters on each dimension zero, the* **`constraints=cases`** *argument of the* **`set`** *command can be used to have the mean of each latent dimension set to zero as an alternative constraint. If this were done, all item parameters would be estimated, but the mean of each of the latent dimensions would be zero.*

Figure 10 is a map of the five latent dimensions and the item difficulties. For the purposes of this figure, we have omitted the rightmost panel, which shows the item step-parameter estimates.

```
===========================================================================================
Australian Performance Assessment Data                      Thu Dec 19 17:26:55
MAP OF LATENT DISTRIBUTIONS AND RESPONSE MODEL PARAMETER ESTIMATES
===========================================================================================
                       Dimension                              Terms in the Model Statement
--------------------------------------------------
          1         2         3         4         5                    +item
-------------------------------------------------------------------------------------------
   5           |                             |         |
               |                             |         |
               |                             |         |
              X|                             |         |
              X|         |        X|         |         |
   4          X|         |        X|         |         |
             XX|         |       XX|         |         |
             XX|         |        X|         |         |
            XXX|         |        X|         |         |
            XXX|         |        X|         |         |25
   3        XXX|        X|      XXX|         |         |
            XXX|        X|      XXX|         |         |
          XXXXX|        X|      XXX|         |         |
          XXXXX|        X|     XXXX|         |         |
          XXXXX|        X|      XXX|         |        X|
   2      XXXXX|     XXXX|    XXXXX|        X|        X|6
          XXXXX|      XXX|    XXXXX|        X|       XX|
         XXXXXX|     XXXX|    XXXXX|        X|       XX|12 13
         XXXXXX|    XXXXX|   XXXXXX|      XXX|       XX|
         XXXXXX|   XXXXXX|    XXXXX|      XXX|      XXX|
   1       XXXX|   XXXXXX|    XXXXX|     XXXX|     XXXX|
          XXXXX|  XXXXXXX|      XXX|  XXXXXXX|     XXXX|17
           XXXX|  XXXXXXX|     XXXX|  XXXXXXX|   XXXXXX|8 11
            XXX|  XXXXXXX|     XXXX| XXXXXXXX| XXXXXXXX|28
           XXXX|  XXXXXXX|    XXXXXXXXXXXXXXX|  XXXXXXX|4 22 23
            XXX|    XXXXX|    XXXXX|XXXXXXXXX|  XXXXXXX|2
   0        XXX|    XXXXX|     XXXX| XXXXXXXX|    XXXXX|14 16
             XX|   XXXXXX|      XXX| XXXXXXXX|   XXXXXX|10 24 26 27
             XX|     XXXX|       XX|  XXXXXXX|   XXXXXX|20
              X|     XXXX|      XXX|   XXXXXX|    XXXXX|3 15 19
              X|      XXX|       XX|   XXXXXX|    XXXXX|5
  -1          X|       XX|       XX|      XXX|     XXXX|1
               |        X|       XX|       XX|     XXXX|18 21
               |        X|       XX|       XX|     XXXX|9
               |        X|        X|        X|      XXX|
               |        X|        X|        X|        X|
  -2           |         |        X|         |        X|
               |         |        X|         |        X|
               |         |         |         |        X|
               |         |         |         |        X|
               |         |         |         |         |7
  -3           |         |         |         |         |
===========================================================================================
Each 'X' represents   5.9 cases
===========================================================================================
```

There are now five estimated latent distributions.

Figure 10 Variable Map for the Five-dimensional Sample Analysis

WITHIN-ITEM AND BETWEEN-ITEM MULTIDIMENSIONALITY

The two preceding sample analyses in this tutorial are examples of what Wang (1995) would call *between-item multidimensionality* (see also Adams, Wilson and Wang (1997)). To assist in the discussion of different types of multidimensional models and tests, Wang introduced the notions of *within-item* and *between-item multidimensionality*. A test is regarded as multi-dimensional between-item if it is made up of several unidimensional subscales. A test is considered multidimensional within-item if any of the items relates to more than one latent dimension.

The Multidimensional Between-item Models

Tests that contain several subscales, each measuring related but distinct latent dimensions, are very commonly encountered in practice. In such tests, each item belongs to only one particular subscale, and there are no items in common across the subscales. In the past, item response modelling of such tests has proceeded by either applying a unidimensional model to each of the scales separately or by ignoring the multidimensionality and treating the test as unidimensional. Both of these methods have weaknesses that make them less desirable than undertaking a joint, multidimensional calibration (Adams, Wilson and Wang, 1997). In the preceding sample analyses in this tutorial, we have illustrated the alternative approach of fitting a multidimensional model to the data.

Multidimensional Within-item Models

If the items in a test measure more than one latent dimension and some of the items require abilities from more than one dimension, then we call the test within-item multidimensional. The distinction between the within-item and between-item multidimensional models is illustrated in Figure 11.

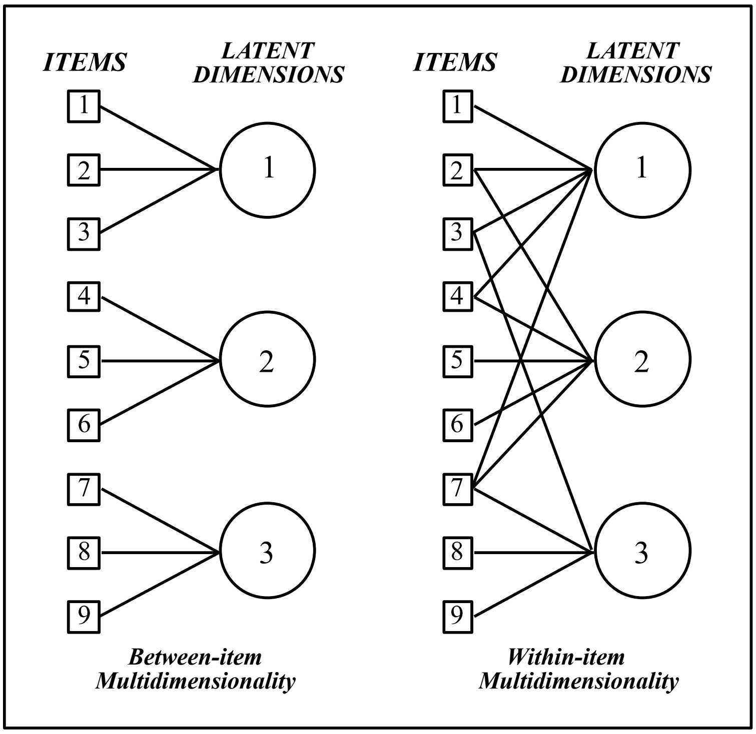

Figure 11 A Graphical Representation of Within-item and Between-item Multidimensionality

In the left of Figure 11, we have depicted a between-item multidimensional test that consists of nine items measuring three latent dimensions. On the right of Figure 11, we have depicted a within-item multidimensional test with nine items and three latent dimensions.

As a final sample analysis in this tutorial, we show how ConQuest can be used to estimate a within-item multidimensional model like that illustrated in Figure 11. For the purpose of this sample analysis, we use simulated data that consist of the responses of 2000 students to nine dichotomous questions. These items are assumed to assess three different latent abilities, with the relationship between the items and the latent abilities as depicted in Figure 11. The generating value for the mean for each of the latent abilities was zero, and the generating covariance between the latent dimensions was:

$$\Sigma = \begin{bmatrix} 1.00 & 0.00 & 0.58 \\ 0.00 & 1.00 & 0.58 \\ 0.58 & 0.58 & 1.00 \end{bmatrix}.$$

The generating item difficulty parameters were -0.5 for items 1, 4 and 7; 0.0 for items 2, 5 and 8; and 0.5 for items 3, 6 and 9.

The files that we use in this sample analysis are:

`ex7c.cqc`	The command statements used to fit the model.
`ex7c.dat`	The data.
`ex7c.prm`	Item parameter estimates.
`ex7c.reg`	Regression coefficient estimates.
`ex7c.cov`	Covariance parameter estimates
`ex7c.shw`	Selected results of the analysis

The contents of the command file necessary for fitting this model are shown in Figure 12. This command file actually runs two analyses. The first is used to obtain an approximate solution that is used as initial values for the second analysis, which is used to produce a more accurate solution.

```
datafile ex7c.dat;
format responses 1-9;
set constraints=cases,update=yes,warnings=no;
score (0,1) (0,1) (   ) (   ) ! items(1);
score (0,1) (0,1) (0,1) (   ) ! items(2);
score (0,1) (0,1) (   ) (0,1) ! items(3);
score (0,1) (0,1) (0,1) (   ) ! items(4);
score (0,1) (   ) (0,1) (   ) ! items(5);
score (0,1) (   ) (0,1) (   ) ! items(6);
score (0,1) (0,1) (0,1) (0,1) ! items(7);
score (0,1) (   ) (   ) (0,1) ! items(8);
score (0,1) (   ) (   ) (0,1) ! items(9);
model items;
export parameters >> ex7c.prm;
export reg_coefficients >> ex7c.reg;
export covariance >> ex7c.cov;
estimate !method=montecarlo, nodes=200, converge=.01;
reset;
datafile ex7c.dat;
format responses 1-9;
set constraints=cases,update=yes,warnings=no;
score (0,1) (0,1) (   ) (   ) ! items(1);
score (0,1) (0,1) (0,1) (   ) ! items(2);
score (0,1) (0,1) (   ) (0,1) ! items(3);
score (0,1) (0,1) (0,1) (   ) ! items(4);
```

Figure12 Command File for Fitting a Within-item Multidimensional Model (continues on next page)

```
26. score (0,1) (   ) (0,1) (   ) ! items(5);
27. score (0,1) (   ) (0,1) (   ) ! items(6);
28. score (0,1) (0,1) (0,1) (0,1) ! items(7);
29. score (0,1) (   ) (   ) (0,1) ! items(8);
30. score (0,1) (   ) (   ) (0,1) ! items(9);
31. model items;
32. import init_parameters << ex7c.prm;
33. import init_reg_coefficients << ex7c.reg;
34. import init_covariance << ex7c.cov;
35. export parameters >> ex7c.prm;
36. export reg_coefficients >> ex7c.reg;
37. export covariance >> ex7c.cov;
38. estimate !method=montecarlo,nodes=1000;
39. show !tables=1:2:3 >> ex7c.shw;
40. quit;
```

Figure 12 Command File for Fitting a Within-item Multidimensional Model (continued from previous page)

1. Read data from the file `ex7c.dat`.
2. The responses are in columns 1 through 9.
3. Set `update` to `yes` and `warnings` to `no` so that current parameter estimates are written to a file at every iteration. This statement also sets `constraints=cases`, which should be used if ConQuest is being used to estimate models that have within-item multidimensionality.

> *EXTENSION:* ***ConQuest can be used to estimate within-item multidimensional models without the use of `constraints=cases`. This will, however, require the user to define his or her own design matrices. A comprehensive description of how to construct design matrices for multidimensional models is beyond the scope of this manual.***

4.-12. These `score` statements describe how the items 'load' on each of the latent dimensions. The first item, for example, has scores on dimension one but not dimensions two or three. The second item is scored on the first and second dimensions, the third on the first and third, and so on.

13. The items are all dichotomous, so we are using the simple logistic model.

> *NOTE:* ***The implicit variable names `item` and `items` are synonymous in ConQuest, so you may use either in ConQuest statements.***

14.-16. The item, regression and covariance parameter estimates will each be written to a file. The combination of the `update` argument in the `set` statement (line 3) and these `export` statements means that these files will be updated at every iteration.

17. In this estimation, we are using the Monte Carlo integration method with 200 nodes and a convergence criterion of 0.01. This analysis is undertaken to provide initial values for the more accurate analysis that follows.

18. Resets all system values so that a new analysis can be undertaken.

19.-31. As for lines 1 through 13.

32.-34. Initial values for all of the parameter estimates are read from the files that were created in the previous analysis.

35.-37. As for lines 14 through 16.

38. The Monte Carlo method of estimation is used with 1000 nodes and the default convergence criterion of 0.001.

39. Tables 1, 2 and 3 are written to `ex7c.shw`.

RUNNING THE WITHIN-ITEM MULTIDIMENSIONAL SAMPLE ANALYSIS

To run this sample analysis, launch the console version of ConQuest by typing the command

```
ConQuestCMD ex7c.cqc
```

ConQuest will begin executing the statements that are in the file `ex7c.cqc`; and as they are executed, they will be echoed on the screen. When ConQuest reaches the `estimate` statement, it will begin fitting a within-item three-dimensional form of Rasch's simple logistic model to the data, using 200 nodes and a convergence criterion of 0.01 with the Monte Carlo method. This analysis will take 14 iterations to converge. ConQuest will then proceed to the second analysis. This analysis begins with the provisional estimates provided by the first analysis and uses 1000 nodes with the default convergence criterion of 0.0001. It takes 345 iterations to converge. The `show` statement at the end of the command file will produce three output tables. The second and third of these are reproduced in Figures 13 and 14. The results in these tables show that ConQuest has done a good job in recovering the generating values for the parameters.

```
===========================================================================
ConQuest: Generalised Item Response Modelling Software  Sun Feb 18 11:14 2007
TABLES OF RESPONSE MODEL PARAMETER ESTIMATES
=====================================================================
TERM 1: items
------------------------------------------------------------------------------
   VARIABLES                          UNWEIGHTED FIT          WEIGHTED FIT
-------------                    ----------------------- -----------------------
     item    ESTIMATE  ERROR^ MNSQ       CI        T  MNSQ       CI        T
------------------------------------------------------------------------------
 1   1        -0.380   0.049  0.99 ( 0.94, 1.06) -0.2 1.00 ( 0.96, 1.04) -0.2
 2   2        -0.009   0.026  1.04 ( 0.94, 1.06)  1.2 1.02 ( 0.95, 1.05)  0.9
 3   3         0.496   0.029  1.03 ( 0.94, 1.06)  1.0 1.03 ( 0.95, 1.05)  1.0
 4   4        -0.529   0.028  1.01 ( 0.94, 1.06)  0.2 1.01 ( 0.94, 1.06)  0.4
 5   5         0.028   0.049  1.00 ( 0.94, 1.06) -0.0 1.00 ( 0.96, 1.04) -0.0
 6   6         0.402   0.050  1.00 ( 0.94, 1.06)  0.1 1.00 ( 0.96, 1.04) -0.0
 7   7        -0.510   0.022  1.03 ( 0.94, 1.06)  0.9 1.00 ( 0.93, 1.07)  0.1
 8   8         0.085   0.049  1.01 ( 0.94, 1.06)  0.2 1.00 ( 0.96, 1.04)  0.3
 9   9         0.528   0.050  1.02 ( 0.94, 1.06)  0.5 1.01 ( 0.96, 1.04)  0.4
---------------------------------------------------------------------
==============================================================================
```

The fit statistics look good (not surprising since the data were simulated to fit this model).

Figure 13 Item Parameter Estimates for a Within-item Three-dimensional Sample Analysis

```
=============================================================================
ConQuest: Generalised Item Response Model                  Feb 18 11:14 2007
TABLES OF POPULATION MODEL PARAMETER ESTI
==============================================================================
REGRESSION COEFFICIENTS

                                                Dimension
                         ---------------------------------------------------
Regression Variable         Dimension 1      Dimension 2      Dimension 3

CONSTANT                   0.000*            0.000*            0.000*
--------------------------------------------------------------------------------
An asterisk next to a parameter estimate indicates that it is constrained
================================================================================
COVARIANCE/CORRELATION MATRIX

                                    Dimension
                         ---------------------------
Dimension                     1        2        3

 Dimension 1                          0.098    0.642
 Dimension 2               0.100               0.580
 Dimension 3               0.667     0.550
---------------------------------------------------
Variance                   0.897     1.077    1.033
---------------------------------------------------
```

The means are all zero because of the constraint used to identify the model.

The estimated covariances are close to their generating values of 0.00 and 0.58.

The estimated variances are close to their generating values of 1.00.

Figure 14 Population Parameter Estimates for a Within-item Three-dimensional Sample Analysis

SUMMARY

In this tutorial, we have seen how ConQuest can be used to fit multidimensional item response models. Models of two, three and five dimensions have been fit.

Some key points covered in this tutorial are:

- The `score` statement can be used to indicate that a multidimensional item response model should be fit to the data.
- The fitting of a multidimensional model as an alternative to a unidimensional model can be used as an explicit test of the fit of data to a unidimensional item response model.
- The secondary analysis of latent ability estimates does not produce results that are equivalent to the 'correct' latent regression results. The errors that can be made in a secondary analysis of latent ability estimates are greater when measurement error is large.
- ConQuest offers two approximation methods, quadrature and Monte Carlo, for computing the integrals that must be computed in marginal maximum likelihood estimation. The quadrature method is generally the preferred approach for problems of three or fewer dimensions, while the Monte Carlo method is preferred for higher dimensions.
- ConQuest can be used to fit models that are multidimensional between-item or multidimensional within-item. Fitting multidimensional within-items requires the use of `constraints=cases`, unless an imported design matrix is used.

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