Importing Design Matrices

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In this tutorial, we provide sample analyses in which the model is described through a design matrix, rather than through a `model` statement. In each of the other sample analyses in this manual, a `model` statement is used to specify the form of the model, and ConQuest then automatically builds the appropriate design matrix. While the `model` statement is very flexible and allows a diverse array of models to be specified, it does not provide access to the full generality of the model that is available when a design matrix is directly specified rather than built with a `model` statement.

Contexts in which the importation of design matrices are likely to be useful include:

Imposing Parameter Equality Constraints: On some occasions, you may wish to constrain the values of one or more item parameters to the same value. For example, you may want to test the hypothesis of the equality of two or more parameters.

Mixing Rating Scales: Under some circumstances, you may need to analyse a set of items that contain subsets of items, each of which use different rating scales. These subsets could be assessing the same latent variable, or they could be assessing different latent variables and a multidimensional analysis may be undertaken.

Mixing Faceted and Non-faceted Data: A set of item responses may include a mix of objectively scored items (for example, multiple choice items) and some items that required the use of raters. Under these circumstances, the rater facet would not apply to the objectively scored items.

Modelling Within-item Multidimensionality: ConQuest can only automatically generate design matrices for within-item multidimensional tests if the mean of the latent variables is set to zero. Within-item multidimensional tests that do not have this constraint can, however, be analysed if a design matrix is imported.

In this tutorial, we will provide two sample analyses in which a design matrix is imported so that a model that cannot be described by a `model` statement can be fitted. The first sample analysis illustrates the use of an imported design to model a mixture of two rating scales. The second shows how within-item multidimensionality without setting the means of the latent variables to zero can be accommodated.

MIXING RATING SCALES

The data we analyse in this section were collected as part of the SEPUP study (Roberts, Wilson and Draney, 1997). It consists of the responses of 721 students to a set of 18 items that used two different rubrics. Items 1, 2, 3, 6, 10, 12, 13, 16, 17 and 18 used one rubric, and items 4, 5, 7, 8, 9, 11, 14, and 15 used an alternative rubric. In this sample analysis, we fit a sequence of three models to these data. First, we fit a rating scale model that imposes a common rating structure on all of the items. Then we use an imported design matrix to fit a model that uses two rating scales, one for the items that used the first rubric and one for the items that used the second rubric. We then fit a partial credit model.

The files used in this sample analysis are:

`ex9a.cqc`	The command statements.
`ex9_1.dat`	The data.

`ex9_1.des`	The design matrix imported to fit the mixture of rating scales.
`ex9_1a.shw`	The results of the rating scale analysis.
`ex9_1b.shw`	The results of the mixture of two rating scales.
`ex9_1c.shw`	The results of the partial credit analysis.

The contents of the command file ex9a.cqc are shown in Figure 1.

```
datafile ex9_1.dat;
format responses 5-9,12,15,17,18,24-32;
codes 1 2 3 4 5;
score (1 2 3 4 5) (0 1 2 3 4);
model item + step;
estimate;
show >> ex9_1a.shw;
reset;
datafile ex9_1.dat;
format responses 5-9,12,15,17,18,24-32;
codes 1 2 3 4 5;
score (1 2 3 4 5) (0 1 2 3 4);
model item + step;
import designmatrix << ex9_1.des;
estimate;
show >> ex9_1b.shw;
reset;
datafile ex9_1.dat;
format responses 5-9,12,15,17,18,24-32;
codes 1 2 3 4 5;
score (1 2 3 4 5) (0 1 2 3 4);
model item + item*step;
estimate;
show >> ex9_1c.shw;
quit;
```

Figure 1 Command File for Fitting a Mixture of Two Rating Scales

1. The data file is `ex9_1.dat`.
2. The `format` statement describes the locations of the 18 items in the data file.
3. The codes 1, 2, 3, 4 and 5 are valid.
4. A `score` statement is used to assign scores to the codes. As this is a undimensional analysis, a `recode` statement could have been used as an alternative to this `score` statement.
5. This `model` statement results in a rating scale model that is applied to all items.
6. Commences the estimation.
7. Writes some results to the file `ex9_1a.shw`.
8. Resets all system values at their defaults so that a new analysis can be started.

9.-12. As for lines 1 through 4 above.

13.-14. These two lines together result in a model being fitted that uses a mixture of two rating scales. The `model` statement must be supplied even when a model is being imported. This `model` statement allows ConQuest to identify the generalised items that are to be analysed with the imported model. In this case, we need ConQuest to identify 18 items, so we simply use a `model` statement that will generate a standard rating scale model for the 18 items. The second line imports the design that is in the file `ex9_1.des`. This matrix will replace the design matrix that is automatically generated by ConQuest in response to the `model` statement. The contents of the imported design are illustrated and described in Figure 2.

15.-17. Estimates the model and writes results to `ex9_1b.shw` and resets the system values.

18.-24. This set of commands is the same as for lines 1 through 7, except that we are fitting a partial credit rather than a rating scale model and writing to the file `ex9_1c.shw`.

> ***NOTE:*** *The number of rows in the imported design matrix must correspond to the number of rows that ConQuest is expecting. ConQuest determines this using a combination of the* ***`model`*** *statement and an examination of the data. The* ***`model`*** *statement indicates which combinations of facets will be used to define generalised items. ConQuest then examines the data to find all of the different combinations; and for each combination, it finds the number of categories.*
>
> *The best strategy for manually building a design matrix usually involves running ConQuest, using a* ***`model`*** *statement to generate a design matrix, and then exporting the automatically generated matrix, using the* ***`designmatrix`*** *argument of the* ***`export`*** *statement. The exported matrix can then be edited as needed.*

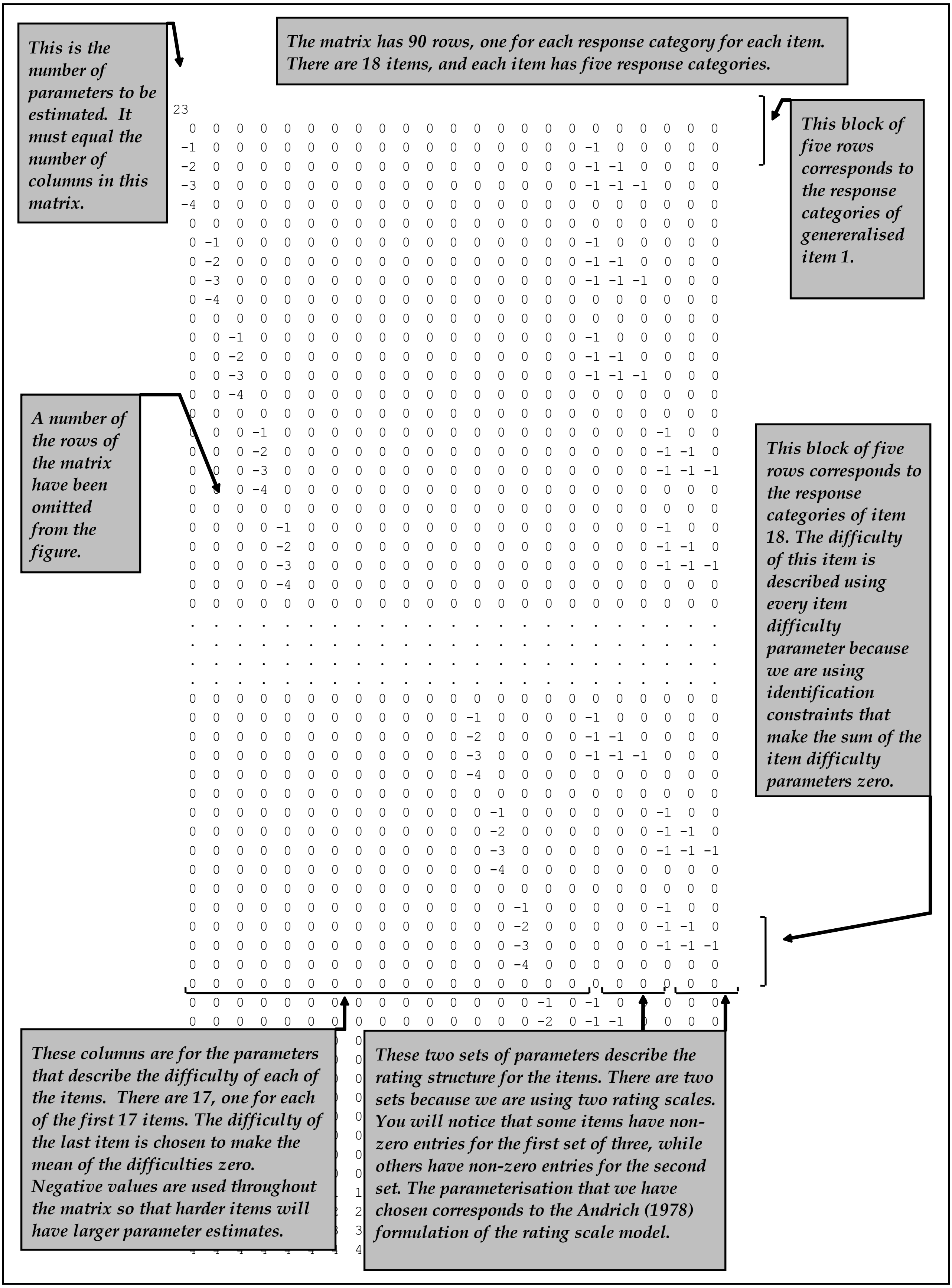

Figure 2 The Imported Design Matrix for Mixing Two Rating Scales

RUNNING THE MIXTURE OF RATING SCALES

To run this sample analysis, launch the console version of ConQuest by typing the command

```
ConQuestCMD ex9a.cqc
```

ConQuest will begin executing the statements that are in the file `ex9a.cqc`; and as they are executed, they will be echoed on the screen. When ConQuest reaches the first `estimate` statement, it will begin fitting the rating scale model to the data. It will take 60 iterations to converge, and the results will be written to the file `ex9_1a.shw`. ConQuest will then proceed to analyse the imported model, taking 78 iterations to converge and writing results to the file `ex9_1b.shw`; and then the partial credit model will be fitted, taking 228 iterations and writing the results to `ex9_1c.shw`.

In Table 1, the fit of this sequence of models is compared using the deviance statistic. Moving from the rating scale to the mixture improves the deviance by 50.42 and requires an additional three parameters; this is clearly significant. The improvement between the mixture and partial credit model is 160.3, and the partial credit model requires 48 additional parameters. This improvement is also significant, although the amount of improvement per parameter is considerably less than that obtained in moving from the rating scale to the mixture of two rating scales. An examination of the parameter fit statistics in the files `ex9_1a.shw`, `ex9_1b.shw` and `ex9_1c.shw` leads to the same conclusions as does the examination of Table 1.

Table 1 Deviance Statistics for the Three Models Fitted to the SEPUP Data

			Difference	
Model	*Deviance*	*Estimated Parameters*	*Deviance*	*Estimated Parameters*
Rating Scale Model	9380.95	22		
Mixture of Two Rating Scales	9330.50	25	50.42	3
Partial Credit Model	9170.22	73	160.28	48

When a model is imported, the ConQuest output will only be provided in an abbreviated form with all parameters listed in one Table. The output produced for the mixture of rating scales is shown in Figure 3.

WITHIN-ITEM MULTIDIMENSIONALITY

As a second sample analysis that uses an imported design matrix, we will return to the within-item multidimensional sample analysis that was used in tutorial 7. In tutorial 7, we used `constraints=cases`, since this enabled ConQuest to automatically generate a design matrix for the model. If the model is to be identified by applying constraints to the item parameters, then ConQuest cannot automatically generate the design matrix for within-item multidimensional models.[1]

[1] This would be necessary if a latent regression model were being estimated.

```
==============================================================================
Parameter Estimates
------------------------------------------------------------------------------
   VARIABLES                          UNWEIGHTED FIT          WEIGHTED FIT
                                 ---------------------- ----------------------
              ESTIMATE   ERROR^  MNSQ       CI        T   MNSQ       CI        T
------------------------------------------------------------------------------
Parameter 1  -0.01670  0.07419  0.79 ( 0.85, 1.15) -3.0  0.82 ( 0.86, 1.14) -2.7
Parameter 2   0.4161[illegible]  [illegible]  [illegible] ( [illegible]5, 1.15) -4.0  0.79 ( 0.85, 1.15) -3.0
Parameter 3   0.246[illegible]  [illegible]  [illegible] ( [illegible]5, 1.15) -4.5  0.71 ( 0.85, 1.15) -4.3
Parameter 4  -0.0732[illegible]  [illegible]  [illegible] ( [illegible]5, 1.15) -5.0  0.68 ( 0.86, 1.14) -4.9
Parameter 5  -1.73129  [illegible]  [illegible] ( 0.85, 1.15) -3.7  0.75 ( 0.84, 1.16) -3.3
Parameter 6   0.22591  0.06270  1.16 ( 0.88, 1.12)  2.5  1.17 ( 0.88, 1.12)  2.7
Parameter 7   0.53499  0.06487  0.99 ( 0.88, 1.12) -0.1  0.99 ( 0.88, 1.12) -0.2
Parameter 8  -1.47939  0.06376  0.73 ( 0.88, 1.12) -4.9  0.73 ( 0.88, 1.12) -4.9
Parameter 9  -1.55998  0.06390  0.87 ( 0.88, 1.12) -2.3  0.87 ( 0.88, 1.12) -2.3
Parameter 10 -0.56111  0.05987  1.00 ( 0.85, 1.15)  0.1  0.99 ( 0.85, 1.15) -0.1
Parameter 11  0.01046  0.06360  1.12 ( 0.85, 1.15)  1.6  1.08 ( 0.85, 1.15)  1.0
Parameter 12  0.41289  0.07914  0.81 ( 0.85, 1.15) -2.7  0.84 ( 0.85, 1.15) -2.2
Parameter 13  0.34666  0.07870  0.77 ( 0.85, 1.15) -3.3  0.77 ( 0.85, 1.15) -3.2
Parameter 14  1.44429  0.08260  0.78 ( 0.79, 1.21) -2.3  0.85 ( 0.78, 1.22) -1.3
Parameter 15  1.60987  0.08355  0.82 ( 0.79, 1.21) -1.8  0.92 ( 0.78, 1.22) -0.7
Parameter 16 -0.05226  0.07534  0.36 ( 0.79, 1.21) -8.0  0.40 ( 0.78, 1.22) -7.1
Parameter 17 -0.00939  0.07552  0.39 ( 0.79, 1.21) -7.6  0.38 ( 0.78, 1.22) -7.4
Parameter 18 -1.97045  0.05359  1.63 ( 0.90, 1.10) 10.0  1.48 ( 0.89, 1.11)  7.3
Parameter 19 -1.46062  0.05894  1.27 ( 0.90, 1.10)  4.6  1.52 ( 0.88, 1.12)  7.0
Parameter 20  0.70711  0.10329  1.96 ( 0.90, 1.10) 14.3  1.55 ( 0.78, 1.22)  4.3
Parameter 21 -3.08112  0.06452  1.41 ( 0.90, 1.10)  6.9  1.36 ( 0.89, 1.11)  5.7
Parameter 22 -1.61459  0.05710  1.26 ( 0.90, 1.10)  4.6  1.22 ( 0.90, 1.10)  3.9
Parameter 23  0.66223  0.07707  1.36 ( 0.90, 1.10)  6.1  1.32 ( 0.87, 1.13)  4.3
```

The parameters are simply listed by number.

For this particular model, the last six values are step parameters. They don't fit well!

Figure 3 Unlabelled Output that is Produced when a Design Matrix is Imported

The files used in this sample analysis are:

`ex9b.cqc`	The command statements.
`ex7c.dat`	The data.
`ex9b.des`	The design matrix imported to fit the within-item multidimensional model.
`ex9b.prm`	Initial values for the item parameter estimates.
`ex9b.reg`	Initial values for the regression parameter estimates.
`ex9b.cov`	Initial values for the covariance parameter estimates.
`ex9b.shw`	The results of the rating scale analysis.

The command file for this sample analysis is shown in Figure 4. As this command file is very similar to that used in tutorial 7, we will only highlight the differences between the contents of Figures 4 and the control files used in tutorial 7.

```
1.  datafile ex7c.dat;
2.  format responses 1-9;
3.  set update=yes,warnings=no;
4.  score (0,1) (0,1) (   ) (   ) ! items(1);
5.  score (0,1) (0,1) (0,1) (   ) ! items(2);
6.  score (0,1) (0,1) (   ) (0,1) ! items(3);
7.  score (0,1) (0,1) (0,1) (   ) ! items(4);
8.  score (0,1) (   ) (0,1) (   ) ! items(5);
9.  score (0,1) (   ) (0,1) (   ) ! items(6);
10. score (0,1) (0,1) (0,1) (0,1) ! items(7);
11. score (0,1) (   ) (   ) (0,1) ! items(8);
12. score (0,1) (   ) (   ) (0,1) ! items(9);
13. model items;
14. import designmatrix << ex9b.des;
15. export parameters >> ex9b.prm;
16. export reg_coefficients >> ex9b.reg;
17. export covariance >> ex9b.cov;
18. estimate !method=montecarlo, nodes=200, converge=.01;
19. reset;
20. datafile ex7c.dat;
21. format responses 1-9;
22. set update=yes,warnings=no;
23. score (0,1) (0,1) (   ) (   ) ! items(1);
24. score (0,1) (0,1) (0,1) (   ) ! items(2);
25. score (0,1) (0,1) (   ) (0,1) ! items(3);
26. score (0,1) (0,1) (0,1) (   ) ! items(4);
27. score (0,1) (   ) (0,1) (   ) ! items(5);
28. score (0,1) (   ) (0,1) (   ) ! items(6);
29. score (0,1) (0,1) (0,1) (0,1) ! items(7);
30. score (0,1) (   ) (   ) (0,1) ! items(8);
31. score (0,1) (   ) (   ) (0,1) ! items(9);
32. model items;
33. import designmatrix << ex9b.des;
34. import init_parameters << ex9b.prm;
35. import init_reg_coefficients << ex9b.reg;
36. import init_covariance << ex9b.cov;
37. export parameters >> ex9b.prm;
38. export reg_coefficients >> ex9b.reg;
39. export covariance >> ex9b.cov;
40. estimate !method=montecarlo,nodes=1000;
41. show >> ex9b.shw;
42. quit;
```

Figure 4 Command File for the Within-item Three-dimensional Model Using an Imported Design

3. & 22. Note that these `set` statements do not include `constraints=cases`, as did the `set` statements in Figure 9-12 (lines 3 and 21). Thus, the means for the latent dimensions will not be constrained, and identification of the model must be assured through the design for the item parameters. ConQuest cannot automatically generate a correct design for a within-item multidimensional model without `constraints=cases`, so an imported design is necessary.

14. & 33. These `import` statements request that a user-specified design be imported from the file `ex9b.des` to replace the design that ConQuest has automatically generated.[2] The contents of the imported design are shown in Figure 5. A full explanation of how designs can be prepared for within-item multidimensional models is beyond the scope of this manual. The interested reader is referred 'Design Matrices' in Chapter 12 of Wu, Adams, Wilson and Haldane (2007) and to Volodin and Adams (1995).

41. The `show` statement cannot produce individual tables when an imported design matrix is used.

Six parameters can be estimated since there are nine items and three dimensions and one constraint must be applied for each dimension.

```
6
 0  0  0  0  0  0
 1  1  1  0  1  0
 0  0  0  0  0  0
-2  0  0  0  0  0
 0  0  0  0  0  0
 0 -2  0  0  0  0
 0  0  0  0  0  0
 0  0 -2  0  0  0
 0  0  0  0  0  0
 1  0  1  1  1  0
 0  0  0  0  0  0
 0  0  0 -1  0  0
 0  0  0  0  0  0
 0  0  0  0 -3  0
 0  0  0  0  0  0
 0  0  0  0  0 -1
 0  0  0  0  0  0
 0  1  0  0  1  1
```

The difficulty of the first item is constrained as identification for the first dimension.

This coefficient is 3 because the item contributes one score point to each of the three dimensions. The use of this coefficient is arbitrary and does not influence the fit of the model. It was chosen here so that the parameterisation is consistent with the default parameterisation used by ConQuest.

The difficulty of the fifth item is constrained as identification for the second dimension.

The difficulty of the last (ninth) item is constrained as identification for the third dimension.

Figure 5 Design Matrix Used to Fit a Three-dimensional Within-item Model

RUNNING THE WITHIN-ITEM MULTIDIMENSIONAL SAMPLE ANALYSIS WITH AN IMPORTED DESIGN MATRIX

To run this sample analysis, launch the console version of ConQuest by typing the command

```
ConQuestCMD ex9b.cqc
```

ConQuest will begin executing the statements that are in the file `ex9b.cqc`; and as they are executed, they will be echoed on the screen. As with the corresponding sample analysis in tutorial 7, this sample analysis will fit a within-in three-dimensional form of Rasch's simple logistic model, first approximately, using 200 nodes, and then more accurately, using 1000 nodes. The first analysis will converge in 11 iterations and the second in 249.

The results obtained from this analysis are shown in Figure 6.

[2] ConQuest will attempt to build a design for within-item multidimensional models, but this design will be incorrect if `constraints=cases` is not used.

```
============================================
Parameter Estimates
--------------------------------------------------------------------------------
   VARIABLES                            UNWEIGHTED                        D FIT
                                    ---------------                     -------------
             ESTIMATE   ERROR^ MNSQ         CI        T  MNSQ       CI         T
--------------------------------------------------------------------------------
Parameter 1  0.22115  0.02108 0.98 ( 0.94, 1.06) -0.6 0.97 ( 0.94, 1.06) -1.0
Parameter 2  0.52852  0.02243 1.01 ( 0.94, 1.06)  0.5 1.02 ( 0.94, 1.06)  0.6
Parameter 3 -0.29989  0.02193 1.03 ( 0.94, 1.06)  0.8 1.03 ( 0.94, 1.06)  0.8
Parameter 4  0.46137  0.03516 1.00 ( 0.94, 1.06) -0.1 0.99 ( 0.95, 1.05) -0.4
Parameter 5 -0.46936  0.01732 1.01 ( 0.94, 1.06)  0.2 1.01 ( 0.93, 1.07)  0.3
Parameter 6 -0.25165  0.03521 1.01 ( 0.94, 1.06)  0.3 1.01 ( 0.95, 1.05)  0.4
--------------------------------------------------------------------

REGRESSION COEFFICIENTS

                                               Dimension
                         ---------------------------------------------------
Regression Variable          Dimension 1      Dimension 2      Dimension 3

CONSTANT                   0.401 ( 0.021)    0.061 ( 0.023)   -0.334 ( 0.023)
-----------------------------------------------------------------------------------
COVARIANCE/CORRELATION MATRIX

                                  Dimension
                         ---------------------------
Dimension                    1        2        3

 Dimension 1                        0.094    0.637
 Dimension 2               0.095             0.580
 Dimension 3               0.656    0.547
----------------------------------------------------
Variance                   0.909    1.083    1.039
----------------------------------------------------
```

The estimated covariance matrix is consistent with that reported in the matching sample analysis in tutorial 7.

These are the estimated means for the three dimensions.

These are the fit statistics for the six estimated item parameters.

The estimated covariance matrix is consistent with that reported in the matching sample analysis in Chapter 9.

Figure 6 Output from the Three-dimensional Within-item Sample Analysis with Imported Design

EXTENSION: *The multidimensional item response model is written as:*

$$f(\mathbf{x};\xi\,\theta) = \Psi(\theta,\xi)\exp\left[\mathbf{x}'(\mathbf{B}\theta+\mathbf{A}\xi)\right],$$

with $\theta \sim \mathrm{MVN}(\mu,\Sigma)$.

If θ *is rewritten as* $\theta^{*}+\mu$, *with* $\theta^{*} \sim \mathrm{MVN}(0,\Sigma)$, *then it can be shown that two models, one described with the design matrices* **A** *and* **B** *and one described with design matrices* **A*** *and* **B***, *are equivalent if*

$$\mathbf{B}^{*}\mu^{*} + \mathbf{A}^{*}\xi^{*} = \mathbf{B}\mu + \mathbf{A}\xi.$$

A small amount of matrix algebra can be used to show that the results reported in Figure 13 satisfies this condition.

SUMMARY

In this tutorial, we have seen how design matrices can be imported to fit models for which ConQuest cannot automatically generate a correct design. Imported designs can be used to fit models that have equality constraints imposed on parameters, models that involve the mixtures of rating scales, models that require the mixing of faceted and non-faceted data, and within-item multidimensional models that do not set the means of the latent variables to zero.

REFERENCES

Roberts, L., Wilson, M., and Draney, K. 1997. *The SEPUP Assessment System: An Overview*. SEPUP Assessment Project. Berkeley: University of California.

Volodin, N. A., and Adams, R. J. 1995. Identifying and estimating a D-dimensional item response model. Paper presented at the International Objective Measurement Workshop, University of California. April, Berkeley, California.

Wu, M. L., Adams, R. J., Wilson, M. R., Haldane, S.A. (2007). *ACER ConQuest Version 2: Generalised item response modelling software* [computer program]. Camberwell: Australian Council for Educational Research.