Extension: Born on a Tuesday

Problem

'I have two children. One is a boy born on a Tuesday. What is the probability that I have two boys?'

Problem steps

This problem was posed by Gary Foshee at a conference in 2010 to celebrate Martin Gardner's life and achievements. A discussion of it can be found on the Outside the Beltway website.

Step 1

The Thursday problem is too hard to look at cold, so we are going to work up to it. To do that we start with the 'Boy and Girl' problem that was formulated by Martin Gardner in *Scientific American* in 1959. The problem poses two questions:

1. 'Mrs Jones has two children. The older child is a girl. What is the probability that both children are girls?'
2. 'Mrs Smith has two children. At least one of them is a boy. What is the probability that both children are boys?'

Let your students discuss the questions in small groups and report their conclusions.

Step 2

Let's first look at the Jones problem. If Mrs Jones has two children, what possible offspring does she have? We list these below, where Y stands for younger and O for older.

(GY, GO), (GY, BO), (BY, GO), (BY, BO)

We know that Mrs Jones has a GO situation, so Mrs Jones can only have (GY, GO) or (BY, BO). There are two situations and each one is equally likely. So the probability that Mrs Jones has two girls is $\frac{1}{2}$.

Over to Mrs Smith. In this family we know that of the four possibilities above, three of the pairs are possible: (GY, BO), (BY, GO) and (BY, BO). So the probability that both children are boys is $\frac{1}{3}$.

At first glance it seems strange that changing from 'the older' to 'at least one' changes the probability.

Step 3

Our original problem takes the Mrs Smith problem a step further, but we'll avoid it for the moment and turn to coin tossing for a while.

'Two coins are tossed and at least one coin shows a head. What is the probability that the other coin is a head?'

Again, give time for small groups to consider this, including any assumptions that are being made, and then let them report back to the whole class.

This coin problem is just Mrs Smith's problem in disguise. (In fact it's also Bertrand's box problem.) Here the fact that two coins are involved takes the place of there being a girl or a boy.

The possibilities with two coins are (H, H), (H, T), (T, H) and (T, T), where the first coin is the first member of each bracket and the second coin the second member. But (T, T) is not a possibility here as we know that at least one coin is a head. So the probability that both coins are heads is $\frac{1}{3}$.

We show this below using a tabular method that some of your students may prefer.

Table 1: The tabular method

		First coin	
		H	**T**
Second coin	**H**	Two heads	One of each
	T	One of each	Two tails

Step 4

Now we toss two coins and the 5c coin comes up heads. What is the probability that the other coin is a head?

What additional information does the mention of a 5c coin give? Is it irrelevant or does it give you further information?

Provide time for your students to discuss this new twist. If the extra information provides no more information, then we are back at Step 2. But if it does, what does it tell us?

In Australia, we have four coins; their values are 5c, 10c, 20c and 50c. Bearing that in mind, it could look as if we first chose a coin from the four possibilities and tossed that, and then we chose another coin and tossed that. So we have a table of possibilities (Table 2).

(By 5cH, etc., we mean that the coin is a 5c coin and shows a head.)

Table 2: All the possibilities

	Possibilities from other coin			
5cH is from one coin	**5cH**	10cH	20cH	50cH
5cH is from one coin	5cT	10cT	20cT	50cT
5cH is from other coin	**5cH**	10cH	20cH	50cH
5cH is from other coin	5cT	10cT	20cT	50cT

There appear to be 20 possibilities here - but one is repeated. The possibility (5cH, 5cH) occurs twice, as highlighted in the table, so there are actually 19 different possibilities.

Of these, seven show two heads. So the probability we are looking for is $\frac{7}{19}$.

Are your students beginning to see the importance of being sure what has been assumed in probability problems?

Step 5

Now your class is ready to tackle the original problem.

Just to make sure that we are all reading these problems the same way, here are the assumptions made in our interpretation of the two problems:

- Each child is either male or female
- The probability of a child being male or female is $\frac{1}{2}$
- The chances of a child being born on any day is $\frac{1}{7}$
- The gender of one child is independent of the gender of the other.

(We can't highlight enough the need to understand the problem's environment in all probability problems. This underlines why many students - and teachers - have difficulty understanding probability.)

Initially the fact that Thursday is involved seems to have no bearing on the outcome, which is surely a probability of $\frac{1}{2}$. Discuss this with your class, then leave them to think about it for a day or two. They could even look up the problem online to see if they can make any sense of the given answer.

Give them time to work on it and then present what they find. Does everyone agree with the conclusions presented?

Step 6

It is important now to go carefully through all of the possibilities. We do this below and count the number of possibilities of each.

1. The first child is the boy born on a Tuesday, and the second child a girl.
   (7 – one for each day of the week)

2. The first child is the boy born on a Tuesday, and the second child a boy.
   (7 – one for each day of the week)

3. The second child is the boy born on a Tuesday, and the first child a girl.
   (7 – one for each day of the week)

4. The second child is the boy born on a Tuesday, and the first child a boy.
   (7 – one for each day of the week)

Ostensibly, this gives 28 possibilities. However, we have counted two boys born on Tuesday *twice* - so there are only 27 possibilities. Of these, 13 pairs are boys. So the probability is $\frac{13}{27}$.

Ask your students if they believe this. Let them have a full discussion of this.

Step 7

Let's redo this using a table this may help the class to get a better idea of how the problem works.

In Table 3, B and G indicate whether we're talking about a boy or girl, while the second letter or letters indicate the day on which they were born (e.g. BM = Boy/Monday; GSU = Girl/Sunday).

We are only interested in the entries where one of the boys is born on Tuesday, so we mark these cells with an X if they are both boys, and with a Y otherwise.

It turns out that 13 of the cells are marked with X and 14 are marked with Y. That means that 27 cells are marked with an X or a Y - so again, the probability of getting an X is $\frac{13}{27}$.

Table 3: All the possibilities regarding Tuesday

		First child													
		BSU	BM	BT	BW	BTH	BF	BSA	GSU	GM	GT	GW	GTH	GF	GSA
Second child	BSU			X											
	BM			X											
	BT	X	X	X	X	X	X	X	Y	Y	Y	Y	Y	Y	Y
	BW			X											
	BTH			X											
	BF			X											
	BSA			X											
	GSU			Y											
	GM			Y											
	GT			Y											
	GW			Y											
	GTH			Y											
	GF			Y											
	GSA			Y											

Where to from here?

- What if the boy is born in the morning? Your students might find it useful to use the tabular method here.
- What if the boy is born in March?
- It would be well worth reading some of Martin Gardner’s books and trying some of his puzzles.
- Your students might also like to invent some problems of their own.