Extension: Polygonal puzzles

Problem

How far can your students extend the six-circle problem?

Problem steps

Step 1

Have a class discussion about what could be done to extend the six-circle problem.

Step 2

Here we will look at circles that form regular polygons with three circles along each side. Start off with a square (Figure 1).

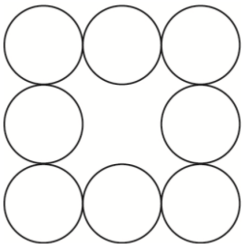

Figure 1: The set up for the eight-circle problem

The eight-circle problem: Can you put the numbers 1 to 8 into the circles above so that the numbers on each side have the same sum?

We won't go step by step through this. Your students have all the techniques they need to solve this problem. Suffice to say that the proof method of Level 1 Step 4.2 gives side sums of 12, 13, 14, 15 and 16. But there don't seem to be any answers for sums of 16. Are the answers in Figure 2 the only ones possible?

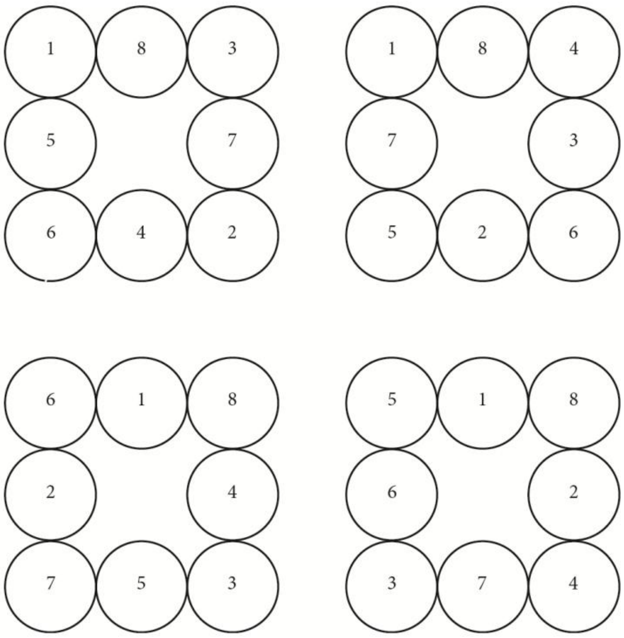

Figure 2: Some answers for the eight-circle problem

It would be useful for the class to go back to the probability method used in Level 1 to see how many answers they might expect.

They should get an experimental probability of about $\frac{8m}{8\times7\times6\times5\times4\times3\times2\times1} = \frac{m}{5040}$ if there are *m* possible answers. This means an awful lot of trials, though.

Step 3

Now try the ten-circle problem, using the pentagon of Figure 3. Can the numbers 1 to 10 be placed in the circles there so that the sum of every set of three numbers on a side add to the same total?

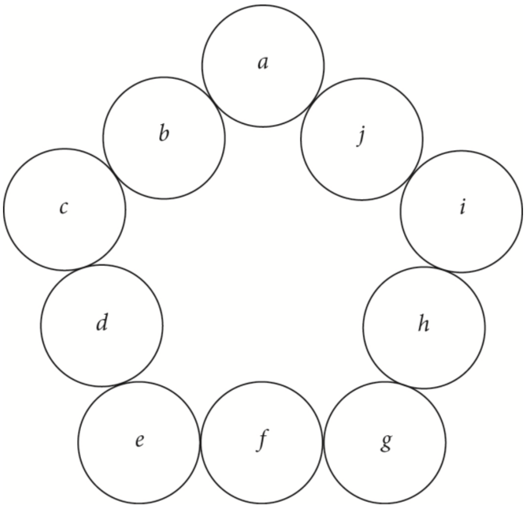

Figure 3: A problem based around a regular pentagon

Step 4

Using the method of Proof 1 in Level 2 Step 3.2, we see that the side sums can only be 14, 15, 16, 17, 18 and 19.

There is at least one answer for the 14 sum. This has *a* = 1, *b* = 9, *c* = 4, *d* = 8, *e* = 2, *f* = 7, *g* = 5, *h* = 6, *i* = 3 and *j* = 10.

What other arrangements can the class find? Do any of the side sums give more than one answer? Do the answers fit any of the patterns of the six-circle problem? Check that the differences between the corner numbers and the opposite middle numbers are not all the same.

Where to from here?

- Can your students find any other problems that are similar to the ones we have been looking at? Can they invent their own problems?
- Do any of these problems work if they replace addition by multiplication?