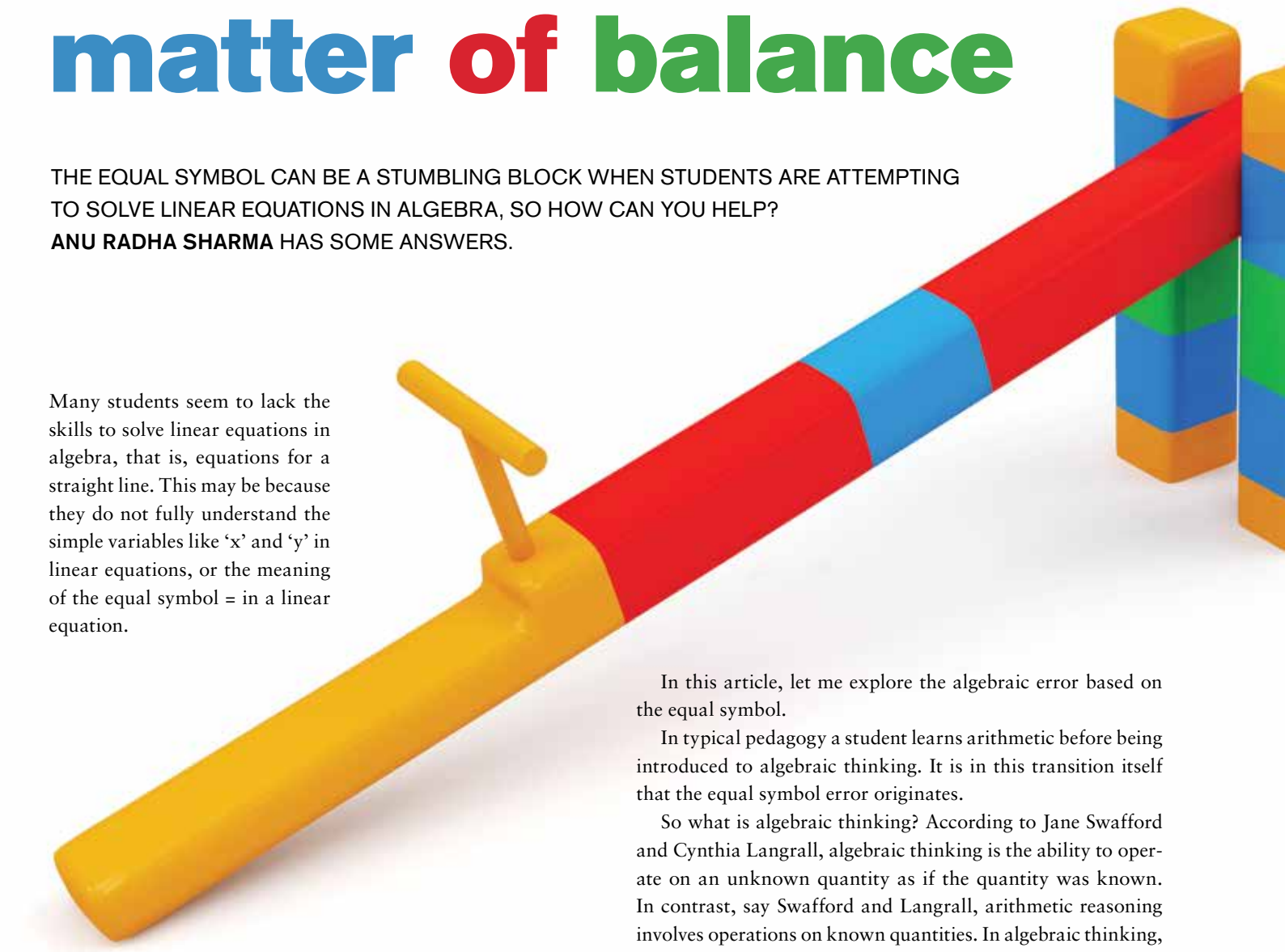
Equal – it's all a matter of balance

THE EQUAL SYMBOL CAN BE A STUMBLING BLOCK WHEN STUDENTS ARE ATTEMPTING TO SOLVE LINEAR EQUATIONS IN ALGEBRA, SO HOW CAN YOU HELP? **ANU RADHA SHARMA** HAS SOME ANSWERS.



Many students seem to lack the skills to solve linear equations in algebra, that is, equations for a straight line. This may be because they do not fully understand the simple variables like 'x' and 'y' in linear equations, or the meaning of the equal symbol = in a linear equation.

The 2014 National Achievement Survey for Class VIII by the National Council for Educational Research and Training found that:

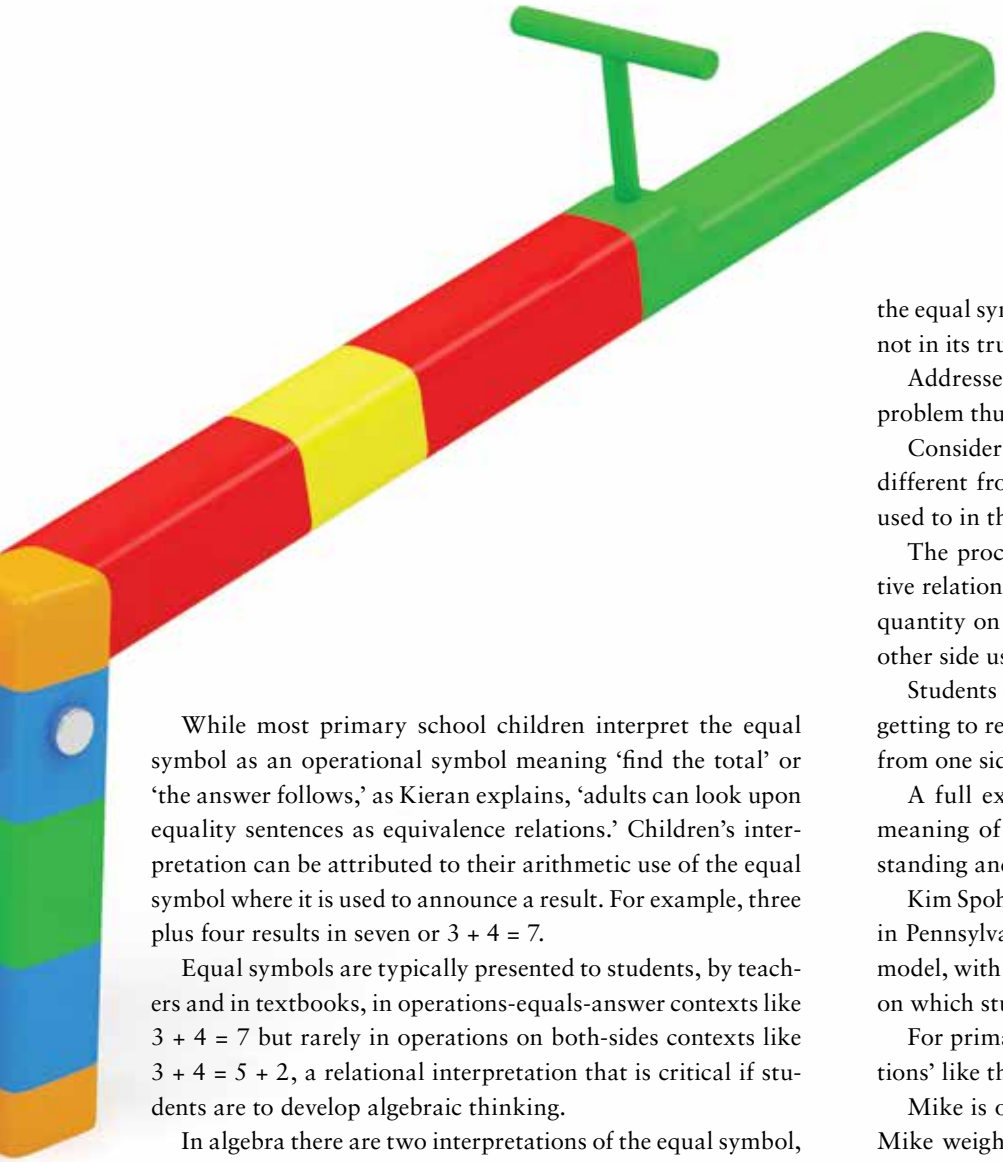
- 58 per cent of students were able to find the numeral coefficient of a monomial.
- 43 per cent of students were able to find the number coefficient of a monomial.
- 78 per cent of students were not able to solve a linear equation in one variable.
- 78 per cent of students could not find the value of an algebraic expression at any integer.

In this article, let me explore the algebraic error based on the equal symbol.

In typical pedagogy a student learns arithmetic before being introduced to algebraic thinking. It is in this transition itself that the equal symbol error originates.

So what is algebraic thinking? According to Jane Swafford and Cynthia Langrall, algebraic thinking is the ability to operate on an unknown quantity as if the quantity was known. In contrast, say Swafford and Langrall, arithmetic reasoning involves operations on known quantities. In algebraic thinking, along with unknown quantities, symbols play an important role.

Arithmetic and algebra share many symbols. One of them is the = symbol of equality. When entering primary school, students come equipped with their own intuitions about basic arithmetic operations. Student's interpretations of symbols and their first symbolisation process in arithmetic are based on these intuitions. That is why, as Carolyn Kieran warns, mathematics teachers need to be careful when interpreting children's symbolism. Simply observing children engaged in reading and writing basic symbolism in arithmetic, we cannot assume that what children understand by that basic symbolism is what we mean by that basic symbolism.



While most primary school children interpret the equal symbol as an operational symbol meaning 'find the total' or 'the answer follows,' as Kieran explains, 'adults can look upon equality sentences as equivalence relations.' Children's interpretation can be attributed to their arithmetic use of the equal symbol where it is used to announce a result. For example, three plus four results in seven or 3 + 4 = 7.

Equal symbols are typically presented to students, by teachers and in textbooks, in operations-equals-answer contexts like 3 + 4 = 7 but rarely in operations on both-sides contexts like 3 + 4 = 5 + 2, a relational interpretation that is critical if students are to develop algebraic thinking.

In algebra there are two interpretations of the equal symbol, the symmetric and the transitive relation. The symmetric relation means that quantities on both sides of the equal symbol are equal, like 3 + 4 = 5 + 2. The transitive relation means that a quantity on one side of the equality symbol can be transferred to the other side using rules.

When used in an equation, the equal symbol indicates that the expressions on the left and right sides balance or have the same value. This can be a stumbling block for students who have learned that the equal symbol means 'the answer follows.'

In most teaching, teachers typically do not explicitly convey these other meanings of the symbol; this may result in errors in solving simple problems involving unknowns. For example, in problems like 4 + 5 = ____ + 7, many students answer 9 (adding the numbers on the left hand side of the equal symbol), or 16 (adding all the known numbers).

In both answers, students perceive the equal symbol to mean 'find the total' or 'the answer follows,' rather than find a relationship between the expressions on the left and right sides.

Sometimes students interpret the equal symbol differently than its actual meaning. For example, consider a word problem,

Sonia purchases three new comic books. She lends five of her comic books to her friends. She is left with four comic books. How many comic books did she have before purchasing the new comic books?

A typical student will solve the problem thus, 5 + 4 = 9 − 3 = 6, violating the symmetric relationship of the equal symbol. Here the equal symbol is used to mark the next step in the procedure, not in its true meaning.

Addressed as a symmetric relation, a student can solve the problem thus, 3 + ____ = 5 + 4.

Consider the process of solving a linear equation. It is quite different from the normal operations students are habitually used to in their primary classes.

The procedure for solving a linear equation uses a transitive relation of the equal symbol. In solving equations often a quantity on one side of the equal symbol is transferred to the other side using rules.

Students often forget the transitive meaning of equality, forgetting to reverse the operation while transforming a quantity from one side of the equation to the other.

A full explanation, demonstration and discussion of the meaning of the equal symbol can enhance students' understanding and ability to solve equations.

Kim Spohn from the Math Institute at Millersville University in Pennsylvania in the United States, suggests using a seesaw model, with the equal symbol being the fulcrum or centre point on which students can balance number sentences.

For primary school students, Spohn suggests 'seesaw equations' like these:

Mike is on one side of the seesaw. Jim is on the other side. Mike weighs 30 kilograms. Jim weighs 20 kilograms. Draw what the seesaw would look like.

Robert and Joey want to play too. Robert weighs 10 kilograms and Joey weighs 20 kilograms. Where should each of them sit to make the seesaw equal? Write a number sentence to show the seesaw is equal. **T**

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The ACER India Subject Consulting Forum aims to provide ongoing learning and professional development through a series of articles in Teacher magazine that deal with common issues and remedial teaching strategies in mathematics, science and English.

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